Math 256 course info

Steven Tschantz

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Basic data

This is course information for

Math 256 - Spring 2010
Mathematical Models in Economics
TTh 2:35-3:50
SC2200 (computer lab)

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Policies

This course is about applying mathematics, defining and experimenting with mathematical models, with particular emphasis on applications in economics. I will assume some basic economics and calculus. Some statistics experience would also be helpful. We will be using the computer program/language *Mathematica* to document the models, program the experiments, and evaluate the results. No background in *Mathematica* is assumed, but an interest in programming is valuable. Initially, we will be spending a good deal of time working through the basics of *Mathematica*, considering simple economic problems and mathematical models

Assignments will be weekly *Mathematica* notebooks to complete and submit by e-mail. There will be a term project where you may work in small teams to research a question, formulate and program a model, presenting your results to the class during the last one or two days of class. There will be no final exam. The term project will count for approximately 25% of your final grade. Class will include lectures on the math and econ, instruction in *Mathematica*, and a significant amount of lab time where you will practice *Mathematica*, working on the assignments, with immediate assistance available. Assignments will be posted Tuesdays, generally to be due the following Tuesday. We will start out slowly, but assignments should soon include more challenging and open ended questions.

I have been involved in part-time economics consulting with Luke Froeb of the Owen graduate school of management here at Vanderbilt, devising and programming mathematical models to help understand various economic questions. This course reflects the kind of work I have been doing. Treat this course as a part-time job. It is important not only that you keep up with posted assignments, but you should go beyond, applying the tools you will learn to better your understanding of questions that interest you.

Sample problem

Suppose a firm has a product it will sell to consumers. How should the firm set the price it will charge for its product? We want a simple model that illustrates the relationship between the price of a product, how much consumers will buy, how much it will cost the firm to produce that much of the product, and what profit the firm will make. Intuitively, the more the firm charges, the less it will be able to sell. It costs more to produce more of the product, though perhaps the added cost of increasing production by some amount may depend on the level of production. In any case, the firm faces a basic tradeoff: increasing the price increases the amount of profit on each unit, but decreases the number units that will be sold.

Suppose the firm sets the price at p. We describe the demand as a function of this variable, $q(p)$, the amount that can be sold at price p each month, say. The firm should know how much raw material and labor it will take to produce a given quantity q, and so will have some idea of the cost $C(q)$ of producing this much product. The profit of the firm is then $\Pi(p) = p\,q(p) - C(q(p))$ each month. We assume that the firm will seek to maximize profit, concluding that the price charged will be determined by the demand and cost functions, $q(p)$ and $C(q)$, and so predicting how price will be affected by changes in demand and cost.

We have a simple model of how firms set prices, expressed as a simple profit maximization problem. Mathematically, we solve for the profit maximizing price using simple calculus. At a critical point,

$$0 = \frac{\partial \Pi(p)}{\partial p} = q(p) + p\,\frac{\partial q(p)}{\partial p} - C'(q(p))\,\frac{\partial q(p)}{\partial p}$$

We can identify important factors in determining the best price. For example, $C'(q)$ is the change in cost per unit change in quantity, what economists call the marginal cost. We might rearrange the terms in this condition to conclude

$$\frac{p - C'(q(p))}{p} = -\left(\frac{\partial q(p)}{\partial p}\,\frac{p}{q(p)}\right)^{-1}$$

the quantity on the right being the profit margin, the profit for each additional unit as a fraction of the price, and the quantity in parentheses on the right being the own price elasticity of demand, the fractional change in demand as a multiple of the fractional change in price.

To solve for the profit maximizing price we would need the functions $q(p)$ and $C(q)$. To get a practical answer for a real world problem we would need to estimate at least the elasticity of demand and marginal cost. To understand how demand and cost determine the optimum price, we can assume simple forms for the demand and cost functions, with parameters that can be adjusted to fit various scenarios. The accuracy of the conclusions drawn will depend on whether the assumed forms are flexible enough and whether the parameters can be estimated with any precision. By stating the assumptions of the model, making explicit the functional forms and parameter values, and solving various scenarios, we can better evaluate the implications of the model in the real world.

To illustrate this problem we can start by assuming a simple linear demand form $q(p) = b + m\,p$ with demand decreasing with increasing price so $m < 0$. We imagine a linear form for cost $C(q) = \mathrm{fc} + \mathrm{mc}\,q$, with some overhead fixed cost fc and constant marginal cost mc. We define these functions and the profit formula in *Mathematica* and solve the corresponding first order conditions. Assuming values for the parameters, we can plot the demand, revenue, cost, and profit as functions of price to help understand how these are determined by the price.

```
In[1]:= quantity[p_] := qintercept + qslope * p;
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In[2]:= cost[q_] := fixedcost + marginalcost * q;

In[3]:= profit[p_] := p * quantity[p] - cost[quantity[p]];

In[4]:= qintercept = 10000.;
        qslope = -100.;
        fixedcost = 25000.;
        marginalcost = 20.;

In[8]:= quantity[p]

Out[8]= 10000. - 100. p

In[9]:= Solve[quantity[p] == 0, p]

Out[9]= {{p → 100.}}

In[10]:= Plot[quantity[p], {p, 0, 100}]
```
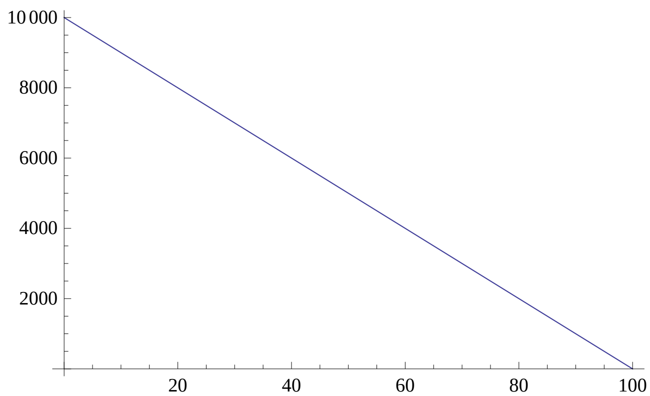
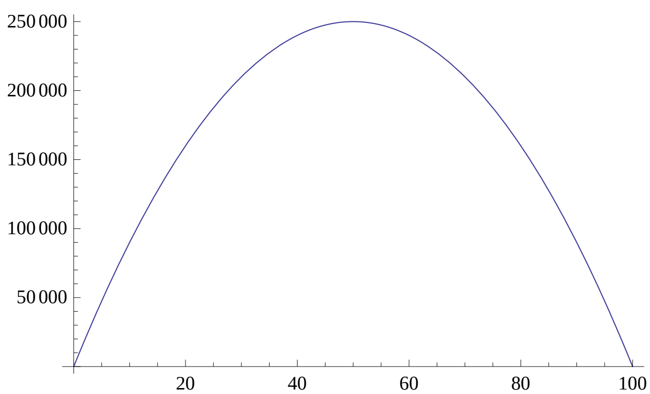

Out[10]=

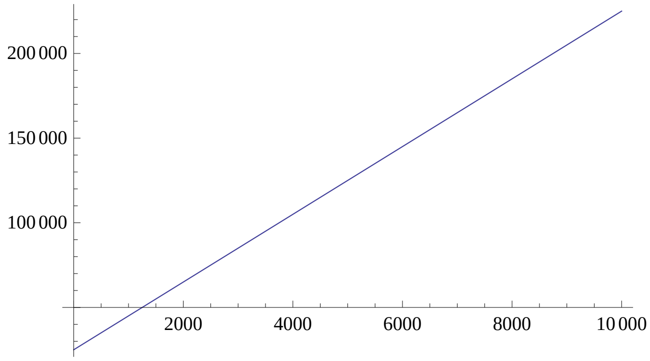



```
In[11]:= Plot[p * quantity[p], {p, 0, 100}]
```

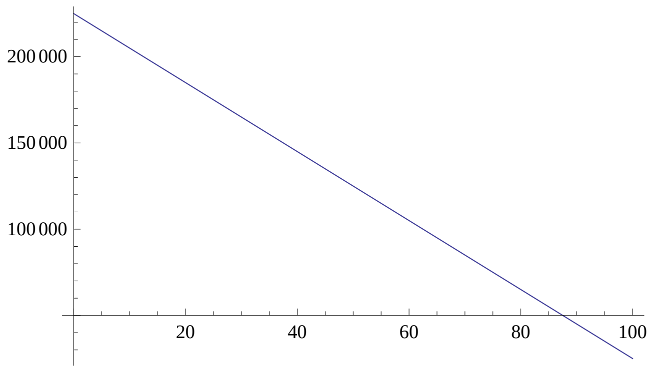
Out[11]=



In[12]:= **Plot[cost[q], {q, 0, 10 000}]**

Out[12]=


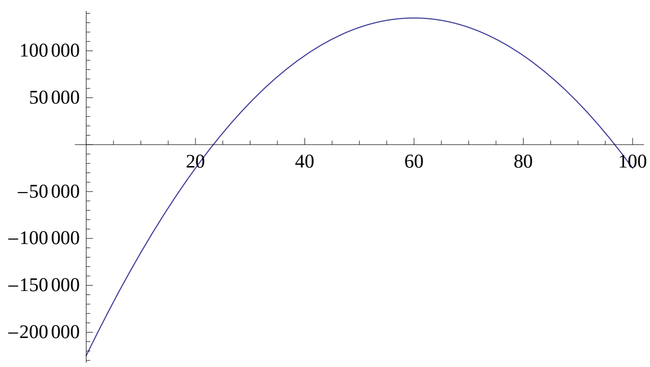
In[13]:= **Plot[cost[quantity[p]], {p, 0, 100}]**

Out[13]=


In[14]:= **profit[p]**

Out[14]= −25 000. − 20. (10 000. − 100. p) + (10 000. − 100. p) p

In[15]:= **Plot[profit[p], {p, 0, 100.}]**

Out[15]=


In[16]:= **Solve[profit[p] == 0, p]**

Out[16]= {{p → 23.2577}, {p → 96.7423}}

In[17]:= **firstordercondition = profit'[p] == 0**

Out[17]= 12 000. − 200. p == 0

In[18]:= **profmaxsoln = Solve[firstordercondition, p]**

Out[18]= {{p → 60.}}

```
In[19]:= profmaxprice = p /. profmaxsoln[[1]]

Out[19]= 60.

In[20]:= profit[profmaxprice]

Out[20]= 135000.

In[21]:= cost[profmaxprice]

Out[21]= 26200.

In[22]:= quantity[profmaxprice]

Out[22]= 4000.

In[23]:= elast = quantity'[profmaxprice] * profmaxprice / quantity[profmaxprice]

Out[23]= -1.5

In[24]:= (profmaxprice - marginalcost) / profmaxprice

Out[24]= 0.666667
```

We can change the parameters and redo the calculations to see how the final price is affected by changes in demand or cost. We can try other functional forms for the demand and cost. If the form is too complex for the program to solve for an exact answer, we can modify the calculation to use numerical root finding instead. With these cases solved, we can expand the model to when the firm sells two products that consumers choose between, to competing firms, to where there are multiple markets with different demands, to cases where production is capacity constrained, and to more complex decisions of firms.

In this course, we will consider a variety of economic questions, describe the mathematical models for understanding these questions, and develop the computational tools needed to solve these models using *Mathematica*.