Math 256
Practice exercise solutions
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Exercises

To get started, as practice with Mathematica, complete the following exercises.

1. Compute 1 + 2 * 3^4 / 5. What precedence is applied to arithmetic operations?

   In[1]:= 1 + 2 * 3^4 / 5
   Out[1]= 167/5

   That is, as in standard mathematical convention, with exponentiation having precedence over multiplication and division, with precedence over addition and subtraction.

   In[2]:= 1 + (2 * (3^4) / 5)
   Out[2]= 167/5

2. Compute 100! (factorial). How many digits are there in this number (please don’t just count)?

   In[3]:= 100!
   Out[3]= 93326215443944152681699238856266700490715968264381621468592963895217599
   \n   99322991560894146397615651828625369792082722375825118521091686400000000
   \n   0000000000000000

   In[4]:= N[Log10[100!]]

   So 100! has 158 digits.
In[5]:= StringLength[ToString[100 !]]
Out[5]= 158

3. Compute the exact value of sine of 90 degrees, 60 degrees, 45 degrees, 30 degrees, and 15 degrees. What are these as decimal approximations? (Note that trig functions take arguments in radians, and Pi is the Mathematica symbol for \( \pi \)).

In[6]:= TableForm[Map[{#, Sin[# Pi / 180], N[Sin[# Pi / 180]]} &, {90, 60, 45, 30, 15}], TableHeadings -> {{}, {"angle x (deg)", "sin(x)", "numerical value"}}]

<table>
<thead>
<tr>
<th>angle x (deg)</th>
<th>sin(x)</th>
<th>numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>0.866025</td>
</tr>
<tr>
<td>60</td>
<td>( \frac{1}{2} )</td>
<td>0.866025</td>
</tr>
<tr>
<td>45</td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>0.707107</td>
</tr>
<tr>
<td>30</td>
<td>( \frac{1}{2} )</td>
<td>0.5</td>
</tr>
<tr>
<td>15</td>
<td>( -\frac{1+\sqrt{3}}{2 \sqrt{2}} )</td>
<td>0.258819</td>
</tr>
</tbody>
</table>

4. Copy and edit the above calculation of effective annual interest rate for interest compounding, changing the calculation from quarterly to daily compounding for 365 periods in a year. Does the effective interest rate depend on the initial principal (except for round-off errors)? (Try the calculation with principal set to an undefined symbol \( p \)).

In[7]:= annualinterestrate = 12.

In[8]:= principal = 10000
Out[8]= 10000

In[9]:= nperiods = 365.

In[10]:= compoundreturn = principal * (1 + annualinterestrate / 100 / nperiods) ^ nperiods
Out[10]= 11274.7

In[11]:= compoundinterest = compoundreturn - principal
The effective annual interest rate is 12.7575\% independent of principal since the interest on the principal, and interest on interest compounding, will all be in proportion to the initial investment.

5. Use Solve to find the intersection points of the line given by the equation \( y=2x-2 \) with a circle of radius 5 centered at the origin (what is the equation of this circle?) as illustrated in the following diagram.

\[
\text{In}[17] = \text{Plot}[2x - 2, \{x, -5, 5\}, \text{Epilog} \rightarrow \{\text{Circle}[[0, 0], 5]\}, \text{AspectRatio} \rightarrow \text{Automatic}] 
\]
In[18]:= Solve[{x^2 + y^2 = 5^2, y = 2 x - 2}, {x, y}]
Out[18]= \{\{x \to -7/5, y \to -24/5\}, \{x \to 3, y \to 4\}\}

In[19]:= Plot[2 x - 2, \{x, -5, 5\},
    Epilog \rightarrow \{Circle[\{0, 0\}, 5], PointSize[0.02], Point[\{x, y\}] /. \}\},
    AspectRatio \rightarrow \text{Automatic}]

6. Define a function by the polynomial \(x^3 - 6 x^2 - 15 x + 10\). Plot the function on the interval \(-10 \leq x \leq 10\). Find the local maxima and minima of this function.

In[20]:= \(f[x_] := x^3 - 6 x^2 - 15 x + 10\)
In[21]:= Plot[f[x], {x, -10, 10}]

Out[21]=

In[22]:= Solve[f'[x] == 0, x]

Out[22]= {{x -> -1}, {x -> 5}}

In[23]:= Plot[f[x], {x, -10, 10}, Epilog -> {PointSize[0.02], Point[{x, f[x]}] / . %}]

Out[23]=