Math 286: MP6 – Due 2/22/14

1) Write a program to solve the following IVP for $0 \leq t \leq T$ and $xl \leq x \leq xr$:

$$u_t + a(t, x)u_x = 0, \quad u(0, x) = u_0(x)$$

with the boundary condition $u(t, 0) = g(t)$ for all $t$.

2) The program should
   a) Input values $nt$ and $nx$ and set $h_x = (xr - xl)/(nx - 1)$ and $h_t = T/(nt - 1)$ and print the value of $a_{max} \ast \lambda$, where $\lambda = h_t/h_x$ and $a_{max}$ is the infinity norm of $a$ over the computational grid.
   
   b) Use the Lax-Friedrichs method on page 17 of the book. Work only on the rectangle $[0, T] \times [xl, xr]$ and use a matrix to store the numerical values. Use the numerical boundary condition (3.4.2) on page 86 of the book at the right hand side. Plot the entire solution as a surface over the rectangle. Also draw a plot with the curves giving the initial function and the solution at $T = 1$.

3) Run your program with $a = 1$ and $xl = -2$, $xr = 3$, $T = 2$ with $u_0(x) = \exp(-10x^2)$ and $g(t) = t$. Make plots for $nx = 101$ and $nt = 31, 41$.

4) Rerun your program with $a(t, x) = t + x$ and $xl = 0$, $xr = 3$, $T = 1$ with $u_0(x) = (1 - x) \ast (x <= 1)$ and $g(t) = 1$. Choose $nx = 201$ and $nt = 271$.

5) Turn in the source code and plots.