Math 226 Homework Problem 1: Due Fri., Jan. 9, 2015

This homework is designed to review elementary linear algebra. Solve all problems by hand, showing work where possible. Do not use a calculator or computer program. Focus on the steps required, and show some work.

1. Find the product

\[
\begin{pmatrix}
1 & 2 & 3 \\
-1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
-2 & 1 \\
0 & 2 \\
1 & 1
\end{pmatrix}.
\]

2. Find the determinants of the following matrices:
   a) \[
   \begin{pmatrix}
   1 & 1 \\
   2 & 2
   \end{pmatrix}.
   \]
   b) \[
   \begin{pmatrix}
   1 & 3 \\
   3 & 1
   \end{pmatrix}.
   \]
   c) \[
   \begin{pmatrix}
   1 & 2 & 3 \\
   0 & 1 & 1 \\
   1 & 0 & 1
   \end{pmatrix}.
   \]

3. Which of the matrices in 2) are nonsingular? Explain why.

4. Solve the system of equations

\[
\begin{align*}
2x + y &= 5 \\
-x + 2y &= 1
\end{align*}
\]

5. Are the vectors \[
\begin{pmatrix}
1 \\
2
\end{pmatrix}, \begin{pmatrix}
2 \\
3
\end{pmatrix} \text{ and } \begin{pmatrix}
-1 \\
1
\end{pmatrix}
\] linearly independent in the space \(\mathbb{R}^2\) of 2-vectors? Explain.

6. Find all eigenvalues and corresponding eigenvectors associated with the matrix in 2b).

7. Find all eigenvalues and corresponding eigenvectors associated with the following matrix:

\[
\begin{pmatrix}
0 & 4 \\
-1 & 0
\end{pmatrix}.
\]