Name (please print):
Math 250 Quiz 3, Friday 31 October 2008, 2 pages, 20 points, 20 minutes

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(10 points) We're going to show that if you add this inference rule

$$A \to C \;\vdash\; (B \to C) \to [(A \vee B) \to C] \qquad (16.3.d.\delta)$$

(as a new assumption) to basic logic (Chapters 13 and 14), then this theorem

$$\vdash A \to (B \to A) \qquad (16.3.a)$$

becomes provable. (You can look at Chapter 16 if you want to, but I don't think it will help.) **Fill in the justifications in the proof below**; step (4) is already done; all the justifications you need are in sections 13.2–13.18 and 14.1–14.5.

(1)	$B \to [(B \to A) \to A]$	assertion (13.8.a)
(2)	$\{[(B \to A) \to A] \to A\} \to (B \to A)$	(1), →-suffixing detached (13.5.a or 13.4.δ)
(3)	$A \to A$	identity (13.2.b)
(4)	$[(B \to A) \to A] \to \{[A \vee (B \to A)] \to A\}$	(3), 16.3.d.δ
(5)	$[A \vee (B \to A)] \to \{[(B \to A) \to A] \to A\}$	(4), permut.det. (13.2.c.δ)
(6)	$A \to [A \vee (B \to A)]$	left ∨-intro (14.1.d)
(7)	$A \to (B \to A)$	(6), (5), (2), repeated trans. (13.6.a)

(10 points) We're going to show that if you add the axiom $\vdash A \to (B \to A)$, as a new assumption (16.3.a), to basic logic (Chapters 13 and 14), then this theorem, (16.10), $\vdash (S \to T) \to [(R \wedge S) \to (R \wedge T)]$, becomes provable. (You can look at Chapter 16 if you want to, but I don't think it will help.) **Fill in the justifications in the proof below**; step (2) is already done; all the justifications you need are in sections 13.2–13.18 and 14.1–14.5. Moreover, in those steps where the justification should be an inference rule referring back to earlier steps in the proof, I have *already indicated* which earlier steps in the proof; so in those justifications you merely need to specify *which* inference rule is being used.

(1)	$(R \wedge S) \to R$	left $\wedge$-elim (14.1.b)
(2)	$R \to [(S \to T) \to R]$	16.3.a
(3)	$(R \wedge S) \to [(S \to T) \to R]$	(1),(2), transitivity (13.5.b)
(4)	$S \to [(S \to T) \to T]$	assertion (13.8.a)
(5)	$(R \wedge S) \to S$	right $\wedge$-elim (14.1.c)
(6)	$(R \wedge S) \to [(S \to T) \to T]$	(5),(4), transitivity (13.5.b)
(7)	$(R \wedge S) \to \{[(S \to T) \to R] \wedge [(S \to T) \to T]\}$	(3),(6), mult. conseq. (14.4.a)
(8)	$\{[(S \to T) \to R] \wedge [(S \to T) \to T]\} \to [(S \to T) \to (R \wedge T)]$	$\wedge$-intro (14.1.f)
(9)	$(R \wedge S) \to [(S \to T) \to (R \wedge T)]$	(7), (8), transitivity (13.5.b)
(10)	$(S \to T) \to [(R \wedge S) \to (R \wedge T)]$	(9), permut det (13.2.c.δ)