

12. [17] Find the equation of the tangent line to the following functions at $x = a$:

(a) $y = e^x; a = \ln 3$ $y' = e^x$

3.2.
27.a

Find line through $(\ln 3, 3)$ with slope 3.

tangent line $y - 3 = 3(x - \ln 3)$

3.3.
62.6

(b) $y = g(x); a = 2$ where $f(2) = 2, f'(2) = 3$ and $g(x) = \frac{f(x)}{x-3}$ $g(2) = \frac{f(2)}{2-3} = \frac{2}{-1} = -2$

By quotient rule, $g'(x) = \frac{f'(x)(x-3) - f(x)}{(x-3)^2}$, hence

$$g'(2) = \frac{f'(2)(2-3) - f(2)}{(2-3)^2} = \frac{3(-1) - 2}{1} = -5$$

~~Find~~ Line through $(2, -2)$ with slope -5

tangent line $y + 2 = -5(x - 2)$

(c) $y = \frac{\cos x}{1 - \cos x}; a = \frac{\pi}{3}$

3.4.53

By quotient rule,

$$y' = \frac{(\cos x)'(1 - \cos x) - (\cos x)(1 - \cos x)'}{(1 - \cos x)^2}$$

$$= \frac{(-\sin x)(1 - \cos x) - (\cos x)(\sin x)}{(1 - \cos x)^2} = \frac{-\sin x}{(1 - \cos x)^2}$$

When $x = \frac{\pi}{3}$ then $\cos x = \frac{1}{2}$, $\sin x = \frac{1}{2}\sqrt{3}$, $y = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$,
and $y' = \frac{-\frac{1}{2}\sqrt{3}}{(\frac{1}{2})^2} = -2\sqrt{3}$. Line through $(\frac{\pi}{3}, 1)$
with slope $-2\sqrt{3}$ is

tangent line $y - 1 = -2\sqrt{3}(x - \frac{\pi}{3})$