

~~3.2.21~~ 8. [9] Find and SIMPLIFY the derivative of the following functions

3.2.21 (a) $f(x) = 10x^4 - 32x + e^2$

$$f'(x) = 40x^3 - 32$$

3.3.52 (b) $h(x) = \frac{(x+1)}{x^2 e^x}$ Use product rule or quotient rule. With product rule, $h(x) = (x^{-1} + x^{-2}) e^{-x}$, hence

$$h'(x) = (x^{-1} + x^{-2})'(e^{-x}) + (x^{-1} + x^{-2})(e^{-x})'$$
$$= (-x^{-2} - 2x^{-3})e^{-x} + (x^{-1} + x^{-2})(-e^{-x})$$

$$= e^{-x}(-x^{-1} - 2x^{-2} - 2x^{-3}) = \frac{-(x^2 + 2x + 2)}{x^3 e^x}$$

3.4.59 9. [5] Show that $y = A \sin t + B \cos t$ satisfies the differential equation $y''(t) + y(t) = 0$ for any constants A and B .

$$y' = A \cos t - B \sin t$$

$$y'' = -A \sin t - B \cos t$$

$$y'' + y = -A \sin t - B \cos t + A \sin t + B \cos t = 0$$

3.2.41.a 10. [7] Find all the points on the graph of $f(x) = 2x^3 - 3x^2 - 12x + 4$ at which the tangent line is horizontal.

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x+1)(x-2)$$

vanishes at $x = -1$ and $x = 2$. Compute

$$f(-1) = -2 - 3 + 12 + 4 = 11 \quad \text{and} \quad f(2) = 16 - 12 - 24 + 4 = -16.$$

points

$$\boxed{(-1, 11) \text{ and } (2, -16)}$$

3.5.51.b 11. [6] A thin rod 4 m in length is heated at its midpoint and the ends are held at a constant temperature of 0° . When the temperature reaches equilibrium, the temperature is given by $T(x) = 40x(4-x)$, where $0 \leq x \leq 4$ is the position along the rod. The heat flux at a point on the rod equals $-kT'(x)$ where k is a positive constant. Find the values of x that have positive heat flux and the values of x that have negative heat flux.

$$T(x) = 160x - 40x^2, \quad T'(x) = 160 - 80x,$$

$$\text{heat flux} = -kT'(x) = 80k(x-2)$$

positive heat flux 2 ≤ x ≤ 4

negative heat flux 0 ≤ x ≤ 2