~~[illegible]~~ 7. [14] Use the precise definition of a limit to prove the following limits

2.7.19 (a) $\lim_{x\to 1}(8x+5) = 13$

Let any $\varepsilon > 0$ be given. We shall take $\delta = \varepsilon/8$.

Now suppose some x is given, satisfying $0 < |x-1| < \delta$. Then $|x-1| < \varepsilon/8$, hence $-\varepsilon/8 < x-1 < \varepsilon/8$, hence $-\varepsilon < 8x-8 < \varepsilon$, hence ~~[illegible]~~ $|8x-8| < \varepsilon$, hence $|(8x+5)-13| < \varepsilon$.

QED.

2.7.32 (b) $\lim_{x\to 0}(\frac{1}{x^4} - \sin x) = \infty$

Let any $N > 0$ be given. We shall take $\delta = \frac{1}{\sqrt[4]{N+1}}$.

Now suppose any x is given, satisfying $0 < |x-0| < \delta$. Then $0 < |x| < \frac{1}{\sqrt[4]{N+1}}$, hence $0 < x^4 < \frac{1}{N+1}$, hence $\frac{1}{x^4} > N+1$. Also, $-1 \leq -\sin x \leq 1$. Hence $\frac{1}{x^4} - \sin x > N$. QED.