Have filtration $M \supseteq M_{n-1} \supseteq \cdots \supseteq M_0$ w/ $M_i / M_{i-1} \in \mathcal{B}$.
We just need to show that $J(M) \simeq J(M_{n-1}) \simeq \cdots \simeq J(M_0) = J(0) = *$.

$i : J(M') \hookrightarrow J(M)$; $M' \leq M$

$r : J(M) \longrightarrow J(M')$
$(M_0, M_1) \longmapsto (M_0 \cap M', M_1 \cap M')$

$s : J(M) \longrightarrow J(M)$
$(M_0, M_1) \mapsto (M_0 \cap M', M_1)$.

$$\begin{tikzcd} & M \arrow[r] & M/M' \arrow[dr] & \\ M_1 \arrow[ur, hook] \arrow[r, "\mathrm{id}"] \arrow[rrr, dashed] & M_1 \arrow[r] & M_1/M_0 \arrow[r] & M/M' \times M_1/M_0 \end{tikzcd}$$

$$\boxed{\text{Note: } M_1 \cap M' / M_0 \cap M' \leq M_1 / M_0 \cap M' \leq M_1/M_0 \times M/M'}$$

Why is r order-preserving? (~~…~~ obvious):

$(M_0, M_1) \leq (M_0', M_1') \Rightarrow M_0' \leq M_0 \leq M_1 \leq M_1'$

$\Rightarrow M_0' \cap M' \leq M_0 \cap M' \leq M_1 \cap M' \leq M_1' \cap M'$

$\Rightarrow (M_0 \cap M', M_1 \cap M') \leq (M_0' \cap M', M_1' \cap M')$

and for s? (also obvious)
$(M_0, M_1) \leq (M_0', M_1') \Rightarrow M_0' \leq M_0 \leq M_1 \leq M_1'$

$\Rightarrow$ ~~$M_0 \cap$~~ $M_0' \cap M' \leq M_0 \cap M' \leq M_1 \leq M_1'$

More important: why are the quotients

$$M_1 \cap M' / M_0 \cap M', \quad M_1 / M_0 \cap M' \quad \text{in } \mathcal{B}?$$

and M_1/M_0, M/M' are in $\mathcal{B}$ so their prod is too.

~~Want~~: Know: $ri = id$. Want ~~…~~ $ir \simeq id$. But
$ir \simeq s \simeq id$ b/c
$$(M_0 \cap M', M_1 \cap M') \leq (M_0 \cap M', M_1) \geq (M_0, M_1)$$