TEMPLATE FOR HOMEWORK

YOUR NAME

1.

Theorem 1.1. *Let $(X, \leq)$ be a well ordered set, then the identity map is the only order isomorphism from X to itself.*

If $x \in X$ then we denote by $I_x = \{y \in X \mid y < x\}$, the initial segment of x. Recall that the principle of transfinite induction states that if $A \subset X$ is such that $x \in A$ whenever $I_x \subset A$, then $A = X$.

Lemma 1.2. *Suppose $\theta : X \to X$ is an order isomorphism then $x \leq \theta(x)$ for all $x \in X$.*

Proof. We let $A = \{x \in X \mid x \leq \theta(x)\}$. If $x \notin A$, then we have $x > \theta(x)$, and hence $\theta(x) > \theta(\theta(x))$ and so $\theta(x)$ is a strictly smaller element such that $\theta(x) \notin A$. By contraposition we then have that $x \in A$ whenever $I_x \subset A$, and the principle of transfinite induction then shows that $A = X$. □

Note that for the previous lemma we did not use that θ was surjective. Consider, for example, the map $\mathbb{N} \ni n \mapsto 2n \in \mathbb{N}$.

Proof of Theorem 1.1. Suppose $\theta : X \to X$ is an order isomorphism. Then by Lemma 1.2 we have $x \leq \theta(x)$ for all $x \in X$. Since θ^{-1} is also an order isomorphism, we conclude again from Lemma 1.2 that $\theta(x) \leq \theta^{-1}(\theta(x)) = x$ for all $x \in X$. Hence $x = \theta(x)$ for all $x \in X$. □

2. Environments

"Environments" are commands that are given using the `\begin{}` and `\end{}` syntax. In the preamble, you can see we've defined a bunch of theorem-type environments, including, for example "defn" To get a definition, you type:

Definition 2.1. This is how to define a definition.

Note that the numbering is taken care of automatically, and that we've predefined a bunch of these sorts of environments to take care lemmas, corollaries and such in the header.

Another useful kind of environment is the equation environment. Equations get numbered in sequence with statements, as for example

$$e^{\pi i} + 1 = 0 \tag{2.2}$$

Note if you do not want a numbered equation, you can use the environment "equation*" like so:

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$$e^{\pi i} + 1 = 0$$

You can also typeset math directly in a paragraph by placing it within dollar signs. This is called "math mode." For example: $e^{\pi i} + 1 = 0$. This is useful, but remember that it is harder to read inline math than displayed math.

Remember that letters get put in a different font in math mode, so whenever you are referencing a mathematical object you should always put it in dollar signs. For example, f is a function, but f is just a random letter.

Both [2] and [1] have good lists of other symbols you can use in math mode. These include greek letters ($\alpha, \beta, \Gamma, \Delta, \varepsilon$), operators ($\otimes, +, \sum$) and much more ($\leq, \diamond, \sim$).

3. Arrays and diagrams

Matrices can be a bit tricky. Here's an example:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc \tag{3.1}$$

Here's how you can define a function by cases:

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q}; \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

To draw diagrams, you can use xypic. Here is an example.

$$\begin{array}{ccc} FX & \xrightarrow{Ff} & FY \\ {\scriptstyle \eta_X}\downarrow & & \downarrow{\scriptstyle \eta_Y} \\ GX & \xrightarrow[Gf]{} & GY \end{array} \tag{3.3}$$

References

[1] D. P. Carlisle, and Richard Kaye Essential Mathematical LaTeX2$_\varepsilon$. `http://www4.ncsu.edu/~smsulli2/MA724_Spring2014/el2emath.pdf`

[2] Tobias Oekiter, Hubert Partl, Irene Hyna and Elisabeth Schlegl. The Not So Short Introduction to LaTeX 2$_\varepsilon$. `http://tobi.oetiker.ch/lshort/lshort.pdf`