[redacted]

Chapter 2 Number 17.

$\frac{5}{5}$

17. Seven different gifts are to be distributed among ten children. How many distinct results are possible if no child is to receive more than one gift?

Solution: ∵ no child is to receive more than one gift, the problem actually asks to pick seven children out of ten to and to make an arrangement about their orders.

In the first step, we have $\binom{10}{7}$ possible ways according to combinations.

In the second step, we have $7!$ possible ways according to permutations

∴ combine two steps, we have $\binom{10}{7}7!$ distinct results

Chapter 1. T.E 8. → what are n, m, & r allowed to be?

$\frac{10}{10}$

8. prove. $\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}$

pf: think about the senario about a group consisting of n men and m women. If we want to pick r people out of this group, how many different results are possible

First method: According to combinations, simply pick r people out of $(m+n)$ people, we have $\binom{m+n}{r}$

Second method: we can also split the problem into two steps. The first step is to pick x people out of n men; and the second step is to pick the rest $(r-x)$ people out of m women. $(0 \le x \le r)$

∴ we have $\binom{n}{x}\binom{m}{r-x}$ to denote x men and $(r-x)$ women situation. To combine all possible x, we have

$\binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}$

And $\because$ the first method and the second method give the same result. $\therefore \binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}$

9. Use Theoretical Exercise 8 to prove

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$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$

pf: Spse $m = n = n$, $r = n$ in ex 8. then we have

$$\binom{n+n}{n} = \binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \binom{n}{2}\binom{n}{n-2} + \cdots + \binom{n}{n}\binom{n}{0}$$

$$= \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} \qquad \because \binom{n}{k} = \frac{n!}{(n-k)!\,k!} = \binom{n}{n-k}$$

$$\therefore \binom{n+n}{n} = \binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{k} = \sum_{k=0}^{n} \binom{n}{k}^2$$

13. Show that, for $n > 0$. $\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0$.

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pf: $\because (x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$ Spse $x = (-1)$, $y = 1$. then

$$(-1+1)^n = 0 = \sum_{i=0}^{n} \binom{n}{i} (-1)^i = \sum_{i=0}^{n} (-1)^i \binom{n}{i}$$

$$\therefore \sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0.$$

Chapter 2. problem 3.

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3. Two dices are thrown. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. Describe the events EF, E∪F, FG, EF^c and EFG

Solution: (1) EF. Two dices are thrown, one lands on 1, another lands on 2, 4 or 6. that is EF = {(1,2), (1,4), (1,6), (2,1), (4,1), (6,1)}

12) $E \cup F$: Two dices are thrown, $E \cup F$ means either one lands on 1 and the other could be any value, or the sum of two dices is odd. that is $E \cup F = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,5),(3,4),(3,6),(5,4),(5,6),(2,1),(3,1),(4,1),(5,1),(6,1),(3,2),(5,2),(4,3),(6,3)\}$

FG: Two dices are thrown, one lands on 1 and the other lands on 4. $\therefore FG = \{(1,4),(4,1)\}$

EF^c: F^c means: Two dices are thrown, no dices land on 1. $\therefore EF^c$ means Two dices are thrown, no dice lands on 1 and the sum of two dices is odd. $\therefore$ that is $\{(3,2),(3,4),(3,6),(5,2),(5,4),(5,6),(2,3),(4,3),(6,3),(2,5),(4,5),(6,5)\}$

EFG: Two dices are thrown, one lands on 1 and the other lands on 4. $\therefore EFG = \{(1,4),(4,1)\}$.

$\frac{20}{20}$

10. Sixty percent of the students at a certain school wear neither a ring or a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing.

(a) a ring or a necklace

(b) a ring and a necklace

ordered? unordered?

Solution: The sample space of this problem could be pairs of wearings, let $E = \{(\text{Ring}, x), x \text{ is some wearings}\}$. Let $F = \{(y, \text{necklace}), y \text{ is some wearings}\}$.

The event that wearing either a ring or a necklace is $E \cup F$. And because the student is picked randomly, that means $P(E \cup F) = 1 - P((E \cup F)^c)$ ✓ $\because P((E \cup F)^c) = 0.6$

$\therefore P(E \cup F) = 0.4$

and that means the probability of this student is wearing either a ring or a necklace is 0.4

(b) that is asking us to show $P(E \cap F)$ because $E \cap F$ denotes the outcome that a student wears both a ring and a necklace.

$\because \begin{cases} P(E) = 0.2 \\ P(F) = 0.3 \\ P(E \cup F) = 0.4 \end{cases}$ $\quad \because P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

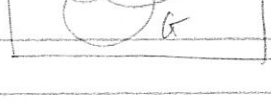
$\therefore P(E \cap F) = P(E) + P(F) - P(E \cup F)$

$= 0.2 + 0.3 - 0.4 = 0.1$

$\therefore$ The probability that this student is wearing a ring and a necklace is 0.1.

chapter 2 Theoretical problem 10.

$\frac{10}{10}$ 10. prove that $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^c F G) - P(E F^c G) - P(E F G^c) - 2P(EFG)$

pf: 

$\because P(E \cup F) = P(E) + P(F) - P(EF)$.

Similarly $P((E \cup F) \cup G) = P(E \cup F) + P(G) - P((E \cup F) G)$

According to the associativity and distribution law. of set union and intersection. $(E \cup F) \cup G = E \cup F \cup G$

and $(E \cup F) G = (EG) \cup (FG)$

$\therefore P(E \cup F \cup G) = P(E \cup F) + P(G) - P((EG) \cup (FG))$

$= P(E) + P(F) + P(G) - P(EF) - P((EG) \cup (FG))$

$\because P((EG) \cup (FG)) = P(EG) + P(FG) - P(EFG)$

$\therefore$ Substitute this into the above equation, we have

$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(EF) - P(EG) - P(FG) + P(EFG)$

$\because P(EF) = P(EFG^c \cup EFG) \quad \because EFG^c$ and EFG are disjoint sets. $\therefore P(EF) = P(EFG^c) + P(EFG)$

Similarly $\begin{cases} P(EG) = P(EF^cG) + P(EFG) \\ P(FG) = P(E^cFG) + P(EFG) \end{cases}$

$\therefore$ substitute again: $\therefore P(E \cup F \cup G) = P(E) + P(F) + P(G) -$
$P(EFG^c) - P(EFG) - P(EF^cG) - P(EFG) - P(E^cFG)$
$- P(EFG) + P(EFG)$

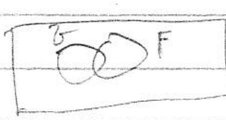
$\therefore P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^cFG) - P(EF^cG) - P(EFG$
$- 2P(EFG)$.

$\frac{10}{10}$ 11. If $P(E) = 0.9$. $P(F) = 0.8$, show that $P(EF) \geq 0.7$. In general prove Bonferroni's inequality, namely
$P(EF) \geq P(E) + P(F) - 1$.

pf: $\because P(E \cup F) = P(E) + P(F) - P(EF)$
$\therefore P(EF) = P(E) + P(F) - P(E \cup F)$ $\because P(E) = 0.9$, $P(F) = 0.8$
and $P(E \cup F) \leq 1$. $\therefore P(EF) \geq 0.9 + 0.8 - 1 = 0.7$.

Similarly $\because P(E \cup F) \leq 1$ $\therefore P(EF) \geq P(E) + P(F) - 1$.

$\frac{10}{10}$ 12. Show that the probability that exactly one of the events E or F occurs equals $P(E) + P(F) - 2P(EF)$

pf: 
exatly one of the events E or F occurs correspond to $E^cF \cup EF^c$

$\therefore P(E^cF \cup EF^c) = P(E^cF) + P(EF^c)$

$\because \begin{cases} P(E^cF \cup EF) = P(E^cF) + P(EF) = P(F) \\ P(EF^c \cup EF) = P(EF^c) + P(EF) = P(E) \end{cases}$

$\therefore P(E^cF \cup EF^c) = P(F) - P(EF) + P(E) - P(EF) = P(E) + P(F) - 2P(EF$

$\frac{10}{10}$

problem A: Consider the $n \times n$ square array of points in the plane $S = \{(i,j) \mid 0 \leq i, j < n\}$ when $n > 1$. Is it possible to draw a path consisting of line segments between the points in this array such that the path meets and changes direction at every point in S, and such that no two line segments making up the path have the same length.

Solution: if for a $n \times n$ square, a line segment in the square can be formed by a pair (x, y) ($0 \leq x \leq (n-1)$, $0 \leq y \leq n-1$, x and y are not 0 at the same time)

If x and y are identical then we have $(n-1)$ different length

✓ If x and y are different, then we have $\frac{\binom{n}{2}}{2} = \frac{n(n-1)}{2}$ different length. In total, we have $(n-1) + \frac{n(n-1)}{2} = \frac{(n+2)(n-1)}{2}$ line segments with different length.

And in order to connect all those points in the square we will need at least (n^2-1) line segments.

$$(n^2-1) - \frac{(n+2)(n-1)}{2} = (n+1)(n-1) - \frac{n+2}{2}(n-1) = (n-1)\left[(n+1) - \frac{n+2}{2}\right]$$

$$= (n-1)\left[n + \cancel{1} - \frac{n}{2} - \cancel{1}\right] = \frac{n}{2}(n-1) \quad \text{for } n \geq 2 \quad \frac{n}{2}(n-1) > 0.$$

$\therefore (n^2-1) > \frac{(n+2)(n-1)}{2}$ $\therefore$ Applying pigeonhole principle there must be two line segments in the path of same length.

great.

Worked with