100

HW1

3. Twenty workers are to be assigned to 20 different jobs, one to each job. How many different assignments are possible?

10/10

Sol: For the first worker, he can be assigned any of the 20 different jobs. For the second worker, his job can be any of the remaining 19 jobs. With the same reasoning, we get $20 \times 19 \times 18 \times \cdots \times 2 \times 1 = 20!$ different assignments.

5. For years, telephone area codes in the United States and Canada consisted of a sequence of three digits. The first digit was an integer between 2 and 9, the second digit was either 0 or 1, and the third digit was any integer from 1 to 9. How many area codes were possible? How many area codes starting with a 4 were possible?

20/20

Sol: (1) There are 8 possible outcomes for the first digit, 2 possible outcomes for the second one and 9 possible outcomes for the third one.

By the basic principle of counting,

$8 \times 2 \times 9 = 144$.

$\therefore$ There are 144 possible area codes.

(2). For area codes starting with a 4, except there is 1 possible outcome for the first digit, the numbers of outcomes for the second and third one are the same.

$1 \times 2 \times 9 = 18$

$\therefore$ 18 area codes starting with a 4 were possible.

10. In how many ways can 8 people be seated in a row if
(a) there are no restrictions on the seating arrangement?

50/50

Sol: The number of ways is the same as the number of permutations of 8 objects.

$\therefore$ There are $8!$ ways.

(b) persons A and B must sit next to each other?

Sol: We can first treat A and B as one person since they always sit together, and so form a group. Then we get 7! ways of arrangement. Within the "A-B" group, there are 2! ways to arrange A and B.

∴ There are 7! × 2! ways of arrangement.

(c) there are 4 men and 4 women and no 2 men or no 2 women can sit next to each other?

Sol: Since no two men or two women can sit next to each other, there are 2 possible arrangements for the two gender:

MFMFMFMF

or FMFMFMFM

with M denoting man and F denoting woman.

For each arrangement, the number of ways males can be seated is the same as the number of permutations of 4 different objects 4!. Similarly, the number of ways females can be seated is 4!.

∴ There is a total number of 4! × 4! × 2 ways they can be seated.

(d) there are 5 men and they must sit next to each other?

Sol: The 5 men form a group as they always sit together. We can treat them as one person. Within the group itself, the 5 men can sit in 5! ways of ordering.

∴ The number of ways the 8 people can be seated is 5! × 4!.

(e) there are 4 married couples and each couple must sit together?

Sol: There are 4! ways the 4 couples can be seated if we treat each couple as one single object. For each couple, there are 2! ways to arrange the 2 married ones.

∴ The total number of ways is 4! × 2! × 2! × 2! × 2!.

13 Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

20/20

Sol: For the 20 people, each one has 20 handshakes. However, A shakes hand with B is the same as B shakes hands with A. Therefore, each handshake is counted twice.

$\therefore \frac{20 \times 19}{2} = 190$ handshakes take place.