Math 247 - Exam 3, November 30, 2011 Name:______________________________

Problem 1 (20 points). Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed normal random variables with mean 5 and variance 2. Let N be a Poisson random variable with parameter 6, which is independent of the X_k's. Compute $E[\Sigma_{k=1}^{N} X_k]$.

Problem 2 (20 points). In each of the following cases determine if the two random variables X and Y are negatively correlated, positively correlated, or have no correlation. Justify your answer.

a) Flip a fair coin 10 times. Let X be the number of heads, and let Y be the number of tails flipped before the first head appears.

b) X and Z are independent and identically distributed, and $Y = X + Z$.

c) Z_1 and Z_2 are identically distributed, $X = Z_1 + Z_2$ and $Y = Z_1 - Z_2$.

Problem 3 (20 points). Consider the experiment of repeatedly rolling a die until a 6 appears. Suppose X is the sum of the values all the rolls, find $E[X]$.

Problem 4 (20 points). Consider an urn containing a large number of coins, and suppose that each of the coins has some probability p of turning up heads when it is flipped. However, this value of p varies from coin to coin. Suppose that the composition of the urn is such that if a coin is selected at random from it, then the p-value of the coin can be regarded as being the value of a random variable that is uniformly distributed over $[0, 1]$. If a coin is selected at random from the urn and flipped twice, compute the probability that both flips result in heads.

Problem 5 (20 points). Let X be and Y be independent exponential random variables with parameter λ. Compute $E[(X+Y)^{10}]$ and simplify your answer.