Math 247 - Exam 2, October 26, 2011 Name: Key

Problem 1 (20 points). Suppose that the travel time in minutes it takes Jacob to get from his home to his probability class is normally distributed with mean 40, and variance 25. If Jacob has a test tomorrow morning and wants to be 95% sure that he is not late for class, how much time before class should he leave his home.

We want to find a s.t.

$$P(X \le a) \ge .95$$

$$P(X \le a) = P\left(\frac{X-40}{5} \le \frac{a-40}{5}\right) = \Phi\left(\frac{a-40}{5}\right)$$

$\therefore \quad \dfrac{a-40}{5} \ge 1.65$ by looking at the chart of Φ.

$\therefore \quad a \ge 48.25$ minutes

Problem 2 (20 points). A company makes computer processors which last (in years) an exponentially distributed time with rate $\lambda = 1/3$. However, 5% of processors the company makes have a defect, and the defected processors last an exponentially distributed time with rate $\lambda = 1/2$. If Laura buys a computer and after 3 years the processor is still working, what is the probability that the processor will continue to work for an additional year?

$$P(3\text{ yrs} \mid 2\text{ yrs}) = \frac{P(3\text{ yrs } \& \text{ } 2\text{ yrs})}{P(2\text{ yrs})} = \frac{P(3\text{ yrs})}{P(2\text{ yrs})}$$

$$= \frac{P(3\text{ yrs} \mid \text{good})P(\text{good}) + P(3\text{ yrs} \mid \text{bad})P(\text{bad})}{P(2\text{ yrs} \mid \text{good})P(\text{good}) + P(2\text{ yrs} \mid \text{bad})P(\text{bad})}$$

$$= \frac{(1-e^{-1})\cdot .95 + (1-e^{3/2})\cdot .05}{(1-e^{-2/3})\cdot .95 + (1-e^{-1})\cdot .05}$$

Problem 3 (20 points). James bought a magic quarter from a street vendor who said that it lands on heads 70% of the times it is flipped. James suspects however that the vendor gave him just a regular quarter. To test this he flips the coin 100 times and finds that it comes up heads 60 of the times. Which is more likely, that James was given the magic quarter, or that James was given a regular quarter. Justify your answer.

If quarter regular & $X_r = \#$ of heads then

$$P(X_r \geq 60) = P\left(\frac{X_r - 50}{100\sqrt{1/4}} \geq \frac{60-50}{100\sqrt{1/4}}\right)$$

$$\approx 1 - \Phi(.2)$$

If quarter magic $X_m = \#$ of heads then

$$P(X_m \leq 60) = P\left(\frac{X_m - 70}{100\sqrt{\frac{7}{10} - \left(\frac{7}{10}\right)^2}} \leq \frac{60-70}{100\sqrt{\frac{7}{10} - \left(\frac{7}{10}\right)^2}}\right)$$

$$\approx \Phi\left(-\frac{1}{\sqrt{21}}\right) = 1 - \Phi\left(\frac{1}{\sqrt{21}}\right)$$

But $.2 \leq \frac{1}{\sqrt{21}}$ since $21 \leq 25$

$\therefore \Phi(.2) \leq \Phi\left(\frac{1}{\sqrt{21}}\right)$

$\therefore P(X_m \leq 60) \leq P(X_r \geq 60)$

So it is more likely a regular quarter.

Problem 4 (20 points). Suppose B, and C are independent random variables which are uniformly distributed over $(0,1)$. Compute the probability that the the equation $x^2 + Bx + C = 0$ has a real root.

From the quadratic formula $x^2 + Bx + C$ has a real root if and only if $B^2 - 4C \geq 0$.

So we want

$$P(4C \leq B^2) = \iint_{\{4x \leq y^2\}} f_{C,B}(x,y)\,dx\,dy$$

$$= \int_0^1 \int_0^{y^2/4} dx\,dy = \int_0^1 \frac{y^2}{4}\,dy = \frac{1}{12}$$

Problem 5 (20 points). Let $X_1, \ldots, X_n$ be independent exponential random variables having a common parameter λ. Determine the distribution function of $\min(X_1, \ldots, X_n)$.

$$F(a) = P(\min(X_1, \ldots, X_n) \leq a)$$
$$= 1 - P(\min(X_1, \ldots, X_n) > a)$$
$$= 1 - P(X_1 > a, X_2 > a, \ldots, X_n > a)$$
$$= 1 - (1 - F_{X_1}(a))^n \quad \text{by independence \& identical distribution}$$

$$= 1 - (e^{-\lambda a})^n = 1 - e^{-\lambda n a}$$

TABLE 5.1: AREA $\Phi(x)$ UNDER THE STANDARD NORMAL CURVE TO THE LEFT OF X

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} \, dy$$