Math 247 - Exam 1, September 28, 2011 Name:_______________________________________

Problem 1 (20 points). Julie has just finished drying her laundry which contained 5 pairs of black socks, 2 pairs of blue socks, 2 pairs of yellow socks, and no other socks. Without looking Julie grabs two socks from the dryer.

a) What is the probability that Julie grabbed a matching pair of socks.

b) Suppose you knew somehow that at least one of the socks Julie grabbed was yellow. Now what is the probability that Julie grabbed a matching pair of socks

Problem 2 (20 points). A certain birth defect occurs in 12% of the population. However, if a family has n children and none of the children have the birth defect, then the probability of the next child born into the family to have the defect goes down to $(12/(n+1))\%$.

The Smith family has 4 children. What is the probability that none of the Smith children have this defect?

Problem 3 (20 points). Prove or give counter-examples to the following statements.

a) For any events E and F, with $P(F) \neq 0$ we have $P(F \mid E \cup F) \geq P(E \mid F)$.

b) For any events E and F, with $P(F) \neq 0$ we have $P(E \mid E \cup F) \geq P(E \mid F)$.

c) For any independent events E and F we have $P(E^c \cup F) - P(F) = P(E \cup F^c) - P(E)$.

Problem 4 (20 points). Suppose X is a discrete random variable whose distribution function is given by

$$F(t) = P(X \leq t) = \begin{cases} 0 & \text{if } t < 0 \\ 1/4 & \text{if } 0 \leq t < 1 \\ 1/2 & \text{if } 1 \leq t < 2 \\ 2/3 & \text{if } 1 \leq t < 3 \\ 1 & \text{if } 3 \leq t \end{cases}$$

a) Compute the expectation and variance of X.

b) Compute the distribution function for the random variable $Y = X^2$.

Problem 5 (20 points). In the city of Nashville, approximately 30 automobiles are stolen every week. Modeling this situation with a Poisson random variable, find the probability that no more than 1 automobile will be stolen in Nashville today.