

*Problem 1.* Find all possible solutions to the following system of linear equations:

$$x + 2y - z + 3w = 0$$

$$x - 4y + 2w = 0$$

$$2y - z + w = 0$$

*Solution 1.* If we look at the associated matrix to this homogeneous system of linear equations then we have

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 1 & -4 & 0 & 2 \\ 0 & 2 & -1 & 1 \end{pmatrix}$$

Subtracting row 1 from row 2 gives

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & -6 & 1 & -1 \\ 0 & 2 & -1 & 1 \end{pmatrix}$$

Adding 3 times row 3 to row 2 gives

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & -2 & 2 \\ 0 & 2 & -1 & 1 \end{pmatrix}$$

Subtracting row 3 from row 1 and then switching rows 2 and 3 gives

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & -2 & 2 \end{pmatrix}$$

Dividing row 3 by -2 and adding it to row 2 gives

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Dividing row 2 by 2 then gives

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

From this we can read off the solution set as:

$$\left\{ \begin{pmatrix} -2t \\ 0 \\ t \\ t \end{pmatrix} \in \mathbb{R}^4 | t \in \mathbb{R} \right\}.$$