Math 196 - Exam 1, September 23, 2008 Name:______________________________

Problem 1 (15 points). Give an explicit solution (or show that one does not exist) for the differential equation given by $y^2 \frac{d^2y}{dx^2} = \frac{dy}{dx}$, under the initial conditions $y(0) = 2$, $\frac{dy}{dx}(0) = \frac{-1}{2}$.

Problem 2 (20 points). Suppose we have a 100 gallon tub which is full with a salt water solution which contains 10 pounds of salt. Suppose we add pure water to the tub at the rate of 1 gallon per minute while we take away from the solution at the same rate.

(1). Assuming that the tub is well stirred, how much salt remains in the pot after 1 hour?

(2). Suppose we repeat the above experiment, but in addition we boil the solution so that (pure) water is evaporated from the tub at the rate of $\frac{1}{10}$ gallons per minute. Then how much salt remains after 1 hour?

Problem 3 (20 points). Give an implicit solution (or show that one does not exist) for the differential equation given by $y(1-x^2)\frac{dy}{dx} = e^{2x}(1+2x-x^2)+xy^2$, under the initial condition $y(0)=2$.

Problem 4 (10 points). The general solution to the differential equation $y''' - 2y'' - y' + 2y = 0$ is given by the function $y = Ae^x + Be^{-x} + Ce^{2x}$, where A, B, and C are constants. Given the initial conditions $y(0) = 4, y'(0) = -1$ and $y''(0) = 1$, determine what the constants A, B, and C are.

Problem 5 (15 points). Give an explicit general solution (or show that one does not exist) for the differential equation given by $\frac{dy}{dx} = \frac{-y}{x} + \frac{1}{e^{xy}}$.

Problem 6 (20 points). Suppose an object is on a flat frictionless slide which is tilted to have slope b, (note in the diagram $b < 0$). Let $X(t)$ and $Y(t)$ be respectively the x-coordinate and y-coordinate of the object at time t.

By Newton's third law of motion we know that the force caused by gravity results in the following diagram:

(1). From this information, derive a second order differential equation relating $X(t)$ to the slope of the slide. (Hint: Using the diagrams above, calculate $\tan\theta$ in multiple ways.)

(2). Suppose that the slide is given by the equation $y = -x + 1$ (in meters), and that the object is placed at rest at the point $(0, 1)$. How long does it take for the object to reach the point $(1, 0)$?

(3). Suppose instead of a flat slide, we have a curve given by the equation $y = 4 - x^2$, give an explicit differential equation for $X(t)$ which describes this situation. (You do not have to solve this differential equation.)