Invariants of continuous fields $C^*$-algebras

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Abstract

I will report on joint work with Ulrich Penning. We show that the automorphism group $\text{Aut}(A)$ of a strongly self-absorbing $C^*$-algebra $A$ is contractible and that the automorphism group $\text{Aut}(A \otimes \mathbb{K})$ of the stabilization of $A$ has the homotopy type of a CW-complex whose homotopy groups we compute. We extend the classification of locally trivial bundles of $C^*$-algebras with compact operators $\mathbb{K}$ as fibers by Dixmier and Douady to the case where the fibers are isomorphic to a stabilized strongly self-absorbing $C^*$-algebra $A \otimes \mathbb{K}$.

We prove that the classifying space for these bundles $B\text{Aut}(A \otimes \mathbb{K})$ has the structure of an infinite loop space with respect to the tensor product. Thus, bundles with fibers $A \otimes \mathbb{K}$ are classified by a generalized continuous cohomology theory which is computable via the Atiyah-Hirzebruch spectral sequence. This allows us to introduce rational characteristic classes for such bundles. A necessary and sufficient $K$-theoretical condition for local triviality is given for continuous fields with fibers $A \otimes \mathbb{K}$ over spaces of finite covering dimension.