

Speaker: Mikael Rørdam

Title: **Classification and strongly self-absorbing C^* -algebras**

Abstract

The program of Elliott to classify (simple and non-simple) C^* -algebras by an invariant that includes K-theory has on the one hand been surprisingly successful in that very large classes of C^* -algebras today are known to be classifiable (as predicted by Elliott), but we also know that not all simple separable nuclear C^* -algebras can be classified. Counterexamples to the classification conjecture were implicitly found in the mid 90s by Villadsen, who gave various examples of C^* -algebras with large dimensional behaviour. These examples were later modified, by the speaker and by Toms, to give explicit counterexamples to the classification conjecture.

Kirchberg and Phillips showed that purely infinite, simple, nuclear separable C^* -algebras (the so-called Kirchberg algebras) in the UCT class are classified by K-theory alone. A simple separable nuclear C^* -algebra is purely infinite if and only if it tensorially absorbs the Cuntz algebra O_∞ . Kirchberg later proved that also non-simple C^* -algebras that tensorially absorb O_∞ can be classified by an ideal related KK-invariant. This result has perhaps the most striking formulation for C^* -algebras that absorb the Cuntz algebra O_2 , where the invariant simply is the primitive ideal space of the C^* -algebra.

Winter and Toms coined the term “strongly self-absorbing” to C^* -algebras D for which there is an isomorphism from D to $D \otimes D$ that is approximately unitarily equivalent to the embedding $d \mapsto 1 \otimes d$. The class of strongly self-absorbing C^* -algebras contain the Cuntz algebras O_∞ and O_2 , the Jiang-Su algebra Z , and also some UHF algebras and their tensor products with O_∞ .

The Jiang-Su algebra Z is a unital, projectionless, infinite dimensional C^* -algebra with a unique trace and which is KK-equivalent to the complex numbers. One can show that the Elliott invariant of an arbitrary (simple) C^* -algebra A and of A tensored with the Jiang-Su algebra coincide if $K_0(A)$ is almost unperforated. More or less all classifiable C^* -algebras are known to absorb the Jiang-Su algebra tensorially. And conversely, there are recent strong results by Winter (and other authors) which show that a C^* -algebra is classifiable if it absorbs the Jiang-Su algebra (under some extra assumptions). The infinite dimensional C^* -algebras mentioned in the first paragraph do not absorb the Jiang-Su algebra.

It therefore makes sense to restrict the classification program to C^* -algebras that absorb tensorially a strongly self-absorbing C^* -algebra, in particular one of O_2 , O_∞ and the Jiang-Su algebra.

In the lecture series we shall discuss these issues more closely.