QUIZ 9

MATH 204

Problem 1. Find an orthonormal basis of $\mathbb{R}^3$ containing $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}), (\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0)$.

Solution. The two vectors are orthogonal and have length 1. The third vector in the orthonormal basis can be found as the cross-product of these two vectors.

Problem 2. Find a basis of the column space of the following matrix:

$$\begin{pmatrix} 2 & 4 & 3 & 6 & 8 \\ 1 & 2 & 3 & 4 & 5 \\ 3 & 7 & 6 & 9 & 12 \end{pmatrix}.$$

Solution. Reduce the matrix to the RREF, find which columns contain pivots. The **corresponding** columns of the original matrix form a basis of the column space.

Problem 3. (a) Suppose that the system of equations $A\vec{x} = \vec{0}$ has 3 pivotal variables, 10 independent variables. What are the rank and the nullity of the matrix A?

Solution. rank is 3, nullity is 10.

(b) Let $A = \begin{pmatrix} 1 & 4 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 12 \end{pmatrix}$. Find a basis of the null-space of A.

Solution. The matrix is in the row echelon form already. The pivotal variables are x_1, x_3, x_4, independent variables are x_2, x_5. The vector form of the solution set is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = x_2 \begin{pmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -8 \\ 0 \\ -5 \\ -12 \\ 1 \end{pmatrix}$$

So a basis of the solution set consists of $\begin{pmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -8 \\ 0 \\ -5 \\ -12 \\ 1 \end{pmatrix}$.

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