

QUIZ 7

MATH 204

Problem 1. Show that the vectors $(1, 2, 3, 4, 5)$, $(1, 3, 1, 1, 1)$, $(2, 1, 0, 0, 3)$ are linearly independent.

Solution. Put the vectors as columns in a 5×3 -matrix, reduce it to the RREF, check that the RREF has three pivotal columns.

Problem 2. Show that the functions x , $3x + 5$, $\sin(x)$ are linearly independent.

Solution. Consider the Wronskian $\det \begin{pmatrix} x & 3x + 5 & \sin(x) \\ 1 & 3 & \cos(x) \\ 0 & 0 & -\sin(x) \end{pmatrix} = 5 \sin(x)$. Since the Wronskian is not 0, the functions are linearly independent.

Problem 3. (a) Suppose that a vector space is spanned by 5 vectors. Can its dimension be 3? Give an argument (if “no”) or an example (if “yes”).

Solution. Yes. Take \mathbb{R}^3 . It has dimension 3, but is spanned, for example, by vectors

$$(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 2, 3), (3, 4, 5).$$

(b) Let P_{2008} be the vector space of polynomials of degree ≤ 2008 . Can 2008 polynomials span P_{2008} ? Explain your answer.

Solution. No, the dimension of P_{2008} is 2009 (the natural basis is $1, x, \dots, x^{2008}$), so by the theorem about dimension, no fewer than 2009 polynomials can span that space.