QUIZ 6

MATH 204

Problem 1. Let θ be a real number. Find the inverse of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix}$ using the adjoint matrix.

Solution. Since $\sin(\theta)^2 + \cos(\theta)^2 = 1$, the determinant of the matrix is 1 (expand along the first row). The adjoint matrix is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$. Since determinant is 1, the adjoint is the same as the inverse.

Problem 2. For which values of a is the following matrix $\begin{pmatrix} a & a^2 & a \\ 0 & a+1 & 0 \\ a^2 & a & a \end{pmatrix}$ invertible?

Solution. The determinant is equal to $(a+1)(a^2 - a^3) = a^2(a+1)(1-a)$ (expand along the second row). The determinant is *not* equal to0 if and only if $a \neq 0, 1, -1$. **Answer.** $a \neq 0, 1, -1$.

Problem 3. (a) Suppose that A, B are $n \times n$-matrices. Suppose that $\det(A) = 5, \det(B) = 6$. Find $\det(A^{-2}B^t)$ (where B^t is B transpose).

Answer. $\frac{6}{25}$.

(b) Give an example of two 2×2-matrices A, B such that $\det(A+B) \neq \det(A) + \det(B)$.

Answer. For example, $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. Indeed, $\det(A) = \det(B) = \det(A) + \det(B) = 0$ but $\det(A+B) = \det(I_2) = 1$.

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