QUIZ 5

MATH 204

Problem 1. Find the area of the parallelogram ABCD on the plane with the following coordinates of vertices: $A = (0, 0), B = (2, 3), C = (5, 7), D = (3, 4)$.

Solution. The area is the absolute value of the determinant of the matrix $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$.

Answer: 1 (the signed area is -1).

Problem 2. Find the determinant of the following matrix:

$$\begin{pmatrix} 0 & a & 0 & 0 \\ b & 0 & 0 & 0 \\ 0 & 0 & 0 & c \\ 0 & 0 & d & 0 \end{pmatrix}.$$

Solution. Expansion along the first row gives $\det(A) = -a \det(M_{12}) = -a \det \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & c \\ 0 & d & 0 \end{pmatrix}$.

Answer. *abcd*

Problem 3. (a) Suppose that a matrix A is reduced to the identity matrix by the following row operations:

$$(\frac{1}{2}R_1)(R_2 - 5R_1)(R_9 - 14R_1)(\frac{5}{7}R_3)(R_5 + 6R_3)(R_6 \leftrightarrow R_9).$$

Find the determinant of A.

Answer: $-\frac{7}{5} \cdot 2 = -\frac{14}{5}$ (use the properties of determinants related to the row operations).

(b) Suppose that $A^2 = A$ (that is A is an idempotent). What are the possible values of $\det(A)$?

Solution. By a property of determinants, $\det(A^2) = \det(A)^2$. So we have $\det(A)^2 = \det(A)$. Hence $\det(A) = 0$ or $\det(A) = 1$.

Answer: 0 or 1.

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