QUIZ 4

MATH 204

Problem 1. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 11 \\ -1 & 4 \end{pmatrix}$. Find the elementary matrix E such that $EA = B$.

Solution. The matrix B is obtained from A by adding two second rows to the first row. So the elementary matrix is $(R_1 + 2R_2) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

Problem 2. Let $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$. Right A^{-1} as a product of elementary matrices.

Solution. Reduce A to RREF:

$$A \xrightarrow{\frac{1}{2}R_1} \begin{pmatrix} 1 & \frac{1}{2} \\ 3 & 2 \end{pmatrix} \xrightarrow{R_2 - 3R_1} \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix} \xrightarrow{2R_2} \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{pmatrix} \xrightarrow{R_1 - \frac{1}{2}R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

So

$$A^{-1} = (R_1 - \frac{1}{2}R_2)(2R_2)(R_2 - 3R_1)(\frac{1}{2}R_1).$$

Answer:

$$A^{-1} = \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}.$$

Problem 3. (a) Find the inverse of the elementary matrix $(R_5 + 8R_6)$.

Answer. $(R_5 - 8R_6)$.

(b) Suppose that matrix A is the product of elementary matrices $(R_3 + 5R_2)(6R_3)(R_3 \leftrightarrow R_5)$. Represent A^{-1} as a product of elementary matrices.

Answer. $(R_3 \leftrightarrow R_5)(\frac{1}{6}R_3)(R_3 - 5R_2)$.

Date: February 12, 2008.