QUIZ 3

MATH 204

Problem 1. Prove that for every square matrix A, the matrix $A + A^t$ is symmetric.

Proof. Let $B = A + A^t$. We need to show that $B^t = B$. Using the properties of the transpose (that the transpose of a sum is the sum of transposes and that $(A^t)^t = A$, we obtain

$$B^t = (A + A^t)^t = A^t + (A^t)^t = A^t + A = B.$$

$\square$

Problem 2. Let $A = \begin{pmatrix} 1 & 0 & 0 & ... & 0 \\ 0 & 2 & 0 & ... & 0 \\ 0 & 0 & 3 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & n \end{pmatrix}$. Compute A^{-1}.

Solution. Consider the augmented matrix $(A \mid I_n)$. In order to reduce it to RREF, we need just to divide each row by the first non-zero number in that row. Thus $A^{-1} = \begin{pmatrix} 1 & 0 & 0 & ... & 0 \\ 0 & \frac{1}{2} & 0 & ... & 0 \\ 0 & 0 & \frac{1}{3} & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & \frac{1}{n} \end{pmatrix}$.

Problem 3. (a) Find numbers α, β such that the matrix $A = \begin{pmatrix} 2 & \alpha & 3 \\ 5 & -6 & 2 \\ \beta & 2 & 4 \end{pmatrix}$ is symmetric.

Solution. $\alpha = 5, \beta = 3$.

(b) Let $A_{i,j}$ be an $n \times n$-matrix with 1 at the (i, j)-place and zeroes everywhere else (i.e. the (i, j)-*matrix unit*). Find the product $A_{i,j} A^t_{i,j}$.

Solution. We have $A^t_{i,j} = A_{j,i}$. The product $A_{i,j} A_{j,i}$ has only one non-zero entry obtained by multiplying the i-th row of $A_{i,j}$ by the i-th column of $A_{j,i}$ (all other rows in $A_{i,j}$ and all other columns of Aj, i are zero). Hence $A_{i,j} A_{j,i} = A_{i,i}$.

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