

QUIZ 10

MATH 204

Problem 1. In \mathbb{R}^2 , find coordinates of the vector $(5, 6)$ in the basis $(10, 7), (4, 5)$ using linear transformations.

Solution: $A = \begin{pmatrix} 10 & 4 \\ 7 & 5 \end{pmatrix}$, $A^{-1} = \frac{1}{22} \begin{pmatrix} 5 & -4 \\ -7 & 10 \end{pmatrix}$, the coordinates are $A^{-1} \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \frac{1}{22} \begin{pmatrix} 1 \\ 25 \end{pmatrix}$.

Problem 2. Show that the linear transformation $T: P_3 \rightarrow P_3$ that takes $p(x)$ to $xp'(x)$ is not an isomorphism (here p' is the derivative of p).

Solution: The transformation is not surjective. Indeed, for every polynomial $p(x)$, the image $q(x) = T(p(x))$ satisfies $q(0) = 0$. Also the transformation has non-trivial kernel: all constant polynomials. Also the range of T has dimension 3 (it is spanned by $T(x), T(x^2), T(x^3)$, since $T(1) = 0$ while $\dim(P_3) = 4$).

Problem 3. a) Are spaces P_3 and \mathbb{R}^4 isomorphic? (explain)

Answer: Yes, they have the same dimension 4.

b) Are spaces \mathbb{R}^5 and $M_{2,2}$ isomorphic? (explain)

Answer: No, the dimension of the first space is 5 while the dimension of the second space is 4.