# Math 204, Sample Exam 2

March 29, 2010

This a sample exam. You should not expect the actual exam to be a subset of the problems you see here. Do not assume that the problems given here cover every possible "type" of problem. The actual exam will be about the same length.
Answers should be **explained and justified**. You may quote results that are either proven in the book or that we have proven in class (unless the question asks you to prove something). You will **lose credit for making false statements**.

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1. Find an orthonormal basis for the range of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$. Also find the matrix of the projection onto the kernel of $A^T$.

2. (a) If $A$ is a symmetric matrix, then show that range($A$) and ker($A$) are perpendicular.

   (b) If $A$ is an $m \times n$ matrix, then show that the projection onto the range of $A$ is given by $A(A^TA)^{-1}A^T$.

3. Consider an orthonormal basis of $\mathbb{R}^3$ given by $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. Suppose that $T$ is the linear transformation that reflects vectors in the plane spanned by $\vec{v}_1$ and $\vec{v}_2$ and rotates the resulting vector through an angle of 45 degrees about the vector $\vec{v}_3$.

   Find the matrix of $T$ in the basis $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

4. Show that $\mathcal{T}$ the collection of upper-triangular matrices is a vector space with respect to matrix addition and scalar multiplication. Show that $\dim(V) = \dfrac{n(n+1)}{2}$.

5. Suppose that $V$ is a subspace of $\mathbb{R}^n$. Show that $V + V^\perp = \mathbb{R}^n$ and that $V \cap V^\perp = \{\vec{0}\}$.

6. If $P$ is the orthogonal projection onto a subspace $V$, then show that $I - P$ is the projection onto $V^\perp$.