

MATH 204 LINEAR ALGEBRA, HOMEWORK 4

Name:

Due: Friday, February 19th, 2010

You should print these pages out two-sided (if possible) to save paper, money, and trees. Staple your sheets together. Write your answers clearly.

- (1) Let A be an $m \times p$ matrix, and let B be an $p \times n$ matrix. Show that the range of A contains the range of AB . Show that the kernel of B is contained in the kernel of AB . Is the reverse inclusion true in either case.

(2) Consider the matrix $A = \begin{bmatrix} 1 & 1 & 4 & 1 \\ 3 & 0 & 1 & -1 \\ 4 & 1 & 13 & 4 \end{bmatrix}$. Find a basis for the range and a basis for the null space of A .

- (3) Suppose that A is an $n \times n$ matrix. Show that the equation $Ax = b$ has a unique solution for every choice of b if and only if the only solution to $Ax = 0$ is $x = 0$.

(4) Suppose that W is a subspace of \mathbb{R}^n . Define the subspace perpendicular to W by

$$W^\perp = \{x \in \mathbb{R}^n : x \cdot w = 0 \text{ for all } w \in W\}.$$

Show that W^\perp is a subspace of \mathbb{R}^n . If The dimension of W is m , then what is the dimension of W^\perp ? You do not need to prove your answer.

(5) Show that the intersection of the subspaces W and W^\perp is the zero subspace.