Math 170, Section 01, Final Exam, Practice
Monday, December 7th, 2009

Name (PRINT clearly):

You will receive full credit only if you show all work and explain all steps.
**Disclaimer**: This is a practice final exam for Math 170. No claim is being made that it will resemble the actual final exam. In particular, you should not assume that every "type" of problem is listed below.

1. Compute the length of the curve $y = \ln(\sec(x))$ from $x = 0$ to $x = \pi/4$.
2. Find the ares of the surface obtained by rotating the graph of $x = \frac{1}{3}(y^2 + 2)^{3/2}$ about the $x$-axis for $1 \leq y \leq 2$.
3. The masses $m_i$ are located at the points $P_i$, for $i = 1, 2, 3$. Find the moments $M_x$ and $M_y$ and locate the centre of mass $(\overline{x}, \overline{y})$ of the system.

$$m_1 = 6, m_2 = 5, m_3 = 9, P_1 = (1, 5), P_2 = (3, -1), P_3 = (-2, -1).$$

4. Find the centroid of the region bounded by $y = x^2$, $x = y^2$, $x = 0$ and $x = 1$.
5. Suppose that the waiting times in a restaurant line are exponentially distributed with a mean waiting time of 2 minutes.
   (a) Write down the formula for the density function.
   (b) What is the probability that you will have to wait in line for 4 minutes or longer.
   (c) The manager of the restaurant posts the following offer: *If you've been waiting longer than $x$ minutes, then you get a free drink.* The manager does not want to give more than 2% of the customers a free drink. What value of $x$ guarantees this.
6. A chemical reaction can be modelled by the equation

$$\frac{dx}{dt} = k(a - x)(b - x)$$

   where $t$ is time and $x$ is the concentration of a certain chemical $C$.
   (a) Assuming that $a \neq b$ solve the above equation for $x$ in terms of $t$. Assume that initially $C$ is not present.
   (b) Assuming that $a = b$ solve the equation.
   (c) Describe the behavior of the solution as $t \to \infty$ in both the above cases.
7. Newton's law of gravitation states that the force acting on a body of mass $m$ that is $x$ kilometers above the Earth's surface is given by $F = \dfrac{mgR^2}{(x + R)^2}$, where $m$ is the mass of the body, $g$ is a constant, and $R$ is the Earth's radius. Remember that force is the mass times acceleration.
   (a) Using these facts setup a differential equation to model the velocity $v(t)$.
   (b) Solve the differential equation and write $v$ as a function of $x$.
   (c) Describe the behavior of $v$.
8. Solve $xy' = y + x^2 \sin(x)$, $y(\pi) = 0$
9. Suppose that a population is modeled by a logistic equation. Show that the population is growing fastest when the population is half the carrying capacity.
10. Decide whether the following sequence converge. If they converge, then compute the limit. Points will be awarded for reasons not guesses.
   (a) $a_n = \dfrac{3^n}{n!}$;
   (b) $a_n = \ln(n^2 + 1) - 2\ln(n)$;
   (c) $\arctan(n^2 + 87)$;

(d) $a_n = \left(\frac{3n}{1+n}\right)^{n/2}$;

(e) $a_{n+1} = 0.98a_n$, for $n \geq 2$, and $a_1 = 1$.

11. Decide whether the following series converge or diverge. Again reasons not guesses.

    (a) $\sum_{n=1}^{\infty} \frac{n!}{(2n)!} 2^n$

    (b) $\sum_{n=1}^{\infty} \left(\frac{\sqrt{n^2+1}}{2n+3}\right)^n$

    (c) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n+2}$

    (d) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

12. (a) Find the value $1.\overline{316}$ are a ratio of integers.

    (b) Sum the series $\sum_{n=1}^{\infty} (\ln(n+3) - \ln(n))$

13. Consider the following power series $\sum_{n=0}^{\infty} \ln(n+1)x^n$.

    (a) Find the radius of convergence of this power series.

    (b) Find the interval of convergence.

    (c) Let $f$ be the function that is defined by the above power series on its interval of convergence. Find a power series representation of $f'(x)$. What is the radius and interval of convergence of this power series.

    (d) Let $F$ be a function such that $F'(x) = f(x)$ and $F(0) = 1$. Find a power series representation for $F$, give its radius and interval of convergence.

14. Consider the function $f(x) = e^{-x/2}$.

    (a) Find an expression for $f^{(n)}(x)$.

    (b) Write down an expression for the $n$th Taylor polynomial for $f$ centered at the origin.

    (c) Estimate the error in using $T_n$ to approximate $f$ on the interval $[-1, 1]$

    (d) Find a reasonable value of $n$ that guarantees that the approximation from the previous part can be carried out with an error no greater than 0.01.