MATH 170, SECTION 01, EXAM 4, SAMPLE
Monday, November 30th, 2009

Name (PRINT clearly):

You will receive full credit only if you show all work and explain all steps.

1. Find the radius and interval of convergence of the following power series.
   (a) $\sum_{n=1}^{\infty} 2^{n+1}x^n$
   (b) $\sum_{n=1}^{\infty} \frac{n!}{n^n}x^n$
   (c) $\sum_{n=1}^{\infty} \frac{x^n}{2n+1}$
2. Find a power series expansion (centered at the origin) for the function $f(x) = \frac{2+x}{2-x}$. Compute the radius of convergence and determine the interval of convergence of the power series.
3. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n}{1+n^2}x^n$. Let f denote the function defined by this power series on its interval of convergence. Find a power series representation $f'(x)$. Let $F(x)$ denote an anti-derivative of f, find the power series representation for F and f' Compute their radius and interval of convergence.
4. Let $f(x) = \sin(2x)$. Find the 8th degree Taylor polynomial for f centered at the origin. Estimate the error in approximating $\sin(1)$ using this Taylor polynomial. How large a degree is required in order to approximate $\sin(1)$ to within 4 decimal places.