Title: Open problems in clone theory

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Abstract: The main problem in clone theory is to get a description of all clones. In 1941 Emil Post obtained an amazing description of all clones on 2 elements, but in 1959 it was proved that there exists a continuum of clones on \( k \) elements if \( k > 2 \).

The first part of my talk is devoted to the efforts to describe uncountable lattices of clones, in particular, I show how uncountable lattice of all clones of self-dual operations on 3 elements can be described. However, obtaining such description for other maximal clones seems to be too cumbersome and complicated. This motivated me to focus on the following questions: how to check properties of a clone and how to find finite sublattices.

Despite the fact that clone theory has been studied for many years, we still don’t know how to check simple properties of clones. For example we don’t know how to check whether a clone defined by a relation is finitely generated, or how to decide how many clones contain a given operation. In the second part of the talk I try to collect such open problems and present some partial results.