Math 2600/5600 - Linear Algebra - Fall 2015

Extra Problems and Answers/Solutions to Practice Problems for Chapter 4

Note: For some problems answers without full explanation are given, while for some problems complete solutions are given. On homework you are always expected to give complete solutions with full details.

4.1.1. See book for problem. Answers in book.

4.1.2. See book for problem. Answer to (a) in book.

4.1.3. See book for problem. Answer to (a) in book.

4.1.4. See book for problem. Answer to (a) in book.

4.1.11. See book for problem.

Solution: For any a, b, c, d we have

$$\begin{aligned} 0 = \delta \begin{bmatrix} a+c & b+d \\ a+c & b+d \end{bmatrix} &= \delta \begin{bmatrix} a & b \\ a+c & b+d \end{bmatrix} \text{ by (ii)} \quad + \delta \begin{bmatrix} c & d \\ a+c & b+d \end{bmatrix} \text{ by (i)} \\ &= \delta \begin{bmatrix} a & b \\ a & b \end{bmatrix} + \delta \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \delta \begin{bmatrix} c & d \\ a & b \end{bmatrix} + \delta \begin{bmatrix} c & d \\ c & d \end{bmatrix} \text{ by (i)} \\ &= 0 + \delta \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \delta \begin{bmatrix} c & d \\ a & b \end{bmatrix} + 0 \text{ by (ii).} \end{aligned}$$

Hence $\delta \begin{bmatrix} c & d \\ a & b \end{bmatrix} = -\delta \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and in particular $\delta \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = -\delta(I) = -1$ by (iii). Now

$$\begin{aligned} \delta \begin{bmatrix} a & b \\ c & d \end{bmatrix} &= a\,\delta \begin{bmatrix} 1 & 0 \\ c & d \end{bmatrix} + b\,\delta \begin{bmatrix} 0 & 1 \\ c & d \end{bmatrix} \text{ by (i)} \\ &= a\left(c\,\delta \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + d\,\delta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) + b\left(c\,\delta \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + d\,\delta \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right) \text{ by (i)} \\ &= a(c(0) + d(1)) + b(c(-1) + d(0)) = ad - bc = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}. \end{aligned}$$

4.2.20. See book for problem.

Answer: $(-1)i(-1+i) + (2+i)1(3i) + 3(1-i)2 - (-1)(1)(2) - (2+i)(1-i)(-1+i) - 3(i)(3i) = (i+1) + (6i-3) + (6-6i) - (-2) - (4i-2) - (-9) = i + 4 - (4i - 13) = -3i + 17$.

4.2.21. See book for problem. Answer in book.

4.2.22. See book for problem.

Answer: -100.

4.2.25. See book for problem.

Answer: Multiplying any single row by k multiplies the determinant by k. Multiplying the whole matrix by k multiplies all n rows by k, so overall it multiplies the determinant by k^n.

4.2.26. See book for problem.

Answer: From 4.2.25, $\det(-A) = (-1)^n \det A$. So if $\det A = \det -A$, we have $\det A = (-1)^n \det A$ and so $(1-(-1)^n)\det A = 0$. There are two ways for this to be true: if $1-(-1)^n = 0$ or $\det A = 0$. If $1-(-1)^n = 0$, i.e., $(-1)^n = 1$, then either n is even, or the field F that we are working in has characteristic 2. So there are three situations where $\det A = \det(-A)$: (1) n is even, (2) F has characteristic 2, or (3) $\det A = 0$.

4.2.1. See book for problem. Answers in book.

4.2.11. See book for problem. Answer in book.

4.3.1. See book for problem. Answers in book.

4.3.10. See book for problem.

Answer: If $M^k = 0$ then $0 = \det M^k = (\det M)^k$ so $\det M = 0$.

4.3.12. See book for problem.

Answer: If $I = QQ^{\mathrm{T}}$ then $1 = \det I = \det Q \det Q^{\mathrm{T}} = \det Q \det Q = (\det Q)^2$ and so $\det Q = \pm 1$.

4.3.15. See book for problem.

Answer: If A and B are similar then there is invertible Q with $B = Q^{-1}AQ$ so $\det B = \det Q^{-1} \det A \det Q = (\det Q)^{-1} \det A \det Q = \det A$.

4.3.26. See book for problem. Answer in book.