Math 2600/5600 - Linear Algebra - Fall 2015

Extra Problems and Answers/Solutions to Practice Problems for Chapter 2

Note: For some problems answers without full explanation are given, while for some problems complete solutions are given. On homework you are always expected to give complete solutions with full details.

2.1.1. See book for problem. Answers in book.

2.1.9. See book for problem.

Answer: For both (a) and (b) (and actually all other parts of this problem) the easiest thing is to show that scalar multiplication is not preserved.

2.1.20. See book for problem.

Answer: This is quickest using (LTP) and (SSP). Take $\alpha, \beta \in F$.

For $T(V_1)$: (SS1) $0_V \in V_1$ by (SS1) for V_1, so $T0_V = 0_W \in T(V_1)$. (SSP) If $w_1, w_2 \in T(V_1)$ then $w_1 = Tv_1$, $w_2 = Tv_2$ with $v_1, v_2 \in V_1$. Then $\alpha w_1 + \beta w_2 = T(\alpha v_1 + \beta v_2) \in T(V_1)$ because $\alpha v_1 + \beta v_2 \in V_1$ by (SSP) for V_1.

For $T^{-1}(W_1) = \{x \in V \mid Tx \in W_1\}$: (SS1) $T0_V = 0_W \in W_1$ by (SS1) for W_1, so $0_V \in T^{-1}(W_1)$. (SSP) If $v_1, v_2 \in T^{-1}(W_1)$ then $Tv_1, Tv_2 \in W_1$, so $T(\alpha v_1 + \beta v_2) = \alpha Tv_1 + \beta Tv_2$ by (SSP) for W_1, so $\alpha v_1 + \beta v_2 \in T^{-1}(W_1)$.

2.2.1. See book for problem. Answers in book.

2.2.15. See book for problem.

Answer: (a) (SS1) Clearly $0 \in L(V, W)$ satisfies $0(x) = 0$ for all $x \in S$, so $0 \in S^0$. (SSP) Suppose $\alpha, \beta \in F$ and $T_1, T_2 \in S^0$, then for $x \in S$ we have $(\alpha T_1 + \beta T_2)(x) = \alpha T_1(x) + \beta T_2(x) = \alpha \cdot 0 + \beta \cdot 0 = 0$, using the fact that addition and scalar multiplication in $L(V, W)$ are defined pointwise. Hence $\alpha T_1 + \beta T_2 \in S^0$.

2.1.13. See book for problem.

Answer: Suppose $\alpha_1 v_1 + \ldots + \alpha_k v_k = 0$. By applying T we get $\alpha_1 w_1 + \ldots + \alpha_k w_k = 0$. Since $w_1, \ldots, w_k$ is a linearly independent collection, this means $\alpha_1 = \ldots = \alpha_k = 0$.

2.1.19. See book for problem.

Answer: There are many examples. For example, take $T, U : \mathbf{R}^2 \to \mathbf{R}^2$, where T is the identity, and $U(x_1, x_2) = (x_2, x_1)$. Then $R(T) = R(U) = \mathbf{R}^2$ and $N(T) = N(U) = \{0\}$, but $T \neq U$ because $T(1, 2) = (1, 2) \neq U(1, 2) = (2, 1)$.

2.2.13. See book for problem.

Answer: Suppose that T and U are linearly dependent. Then $\alpha T + \beta U = 0$ (in $L(V, W)$), where α, β are not both zero. In fact, we cannot have just one of α, β being nonzero, because we know that $T, U \neq 0$. Thus, $\alpha, \beta \neq 0$. Now since $T \neq 0$, there is some $v \in V$ with $Tv \neq 0$. We then have $0 = 0(v) = (\alpha T + \beta U)v = \alpha Tv + \beta Uv$. Thus, $\alpha Tv = -\beta Uv$, or $T(\alpha v) = U(-\beta v)$. Thus, $T(\alpha v) \in R(T) \cap R(U) = \{0\}$, so $0 = T(\alpha v) = \alpha Tv$, and since $\alpha \neq 0$, dividing by α gives $0 = Tv$, a contradiction. Hence, T and U are linearly independent.

2.1.4. See book for problem. Incomplete answers in book.

Answer: To prove T is a linear transformation, use (LTP). If $A = [a_{ij}], B = [b_{ij}] \in F^{2\times 3}$ and $\alpha, \beta \in F$ we have

$$\begin{aligned}
T(\alpha A + \beta B) &= T\left(\alpha \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \beta \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}\right) \\
&= T \begin{bmatrix} \alpha a_{11} + \beta b_{11} & \alpha a_{12} + \beta b_{12} & \alpha a_{13} + \beta b_{13} \\ \alpha a_{21} + \beta b_{21} & \alpha a_{22} + \beta b_{22} & \alpha a_{23} + \beta b_{23} \end{bmatrix} \\
&= \begin{bmatrix} 2(\alpha a_{11} + \beta b_{11}) - (\alpha a_{12} + \beta b_{12}) & (\alpha a_{13} + \beta b_{13}) + 2(\alpha a_{12} + \beta b_{12}) \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 2\alpha a_{11} - \alpha a_{12} & \alpha a_{13} + 2\alpha a_{12} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2\beta b_{11} - \beta b_{12} & \beta b_{13} + 2\beta b_{12} \\ 0 & 0 \end{bmatrix} \\
&= \alpha \begin{bmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{bmatrix} + \beta \begin{bmatrix} 2b_{11} - b_{12} & b_{13} + 2b_{12} \\ 0 & 0 \end{bmatrix} = \alpha TA + \beta TB.
\end{aligned}$$

2.1.5. See book for problem. Incomplete answers in book.

Answer: To prove T is a linear transformation, use (LTP). If $f(x), g(x) \in P_2(\mathbf{R})$ and $\alpha, \beta \in \mathbf{R}$, we have

$$\begin{aligned}
T(\alpha f(x) + \beta g(x)) &= x(\alpha f(x) + \beta g(x)) + \frac{d}{dx}(\alpha f(x) + \beta g(x)) = x\alpha f(x) + x\beta g(x) + \alpha f'(x) + \beta g'(x) \\
&= \alpha(xf(x) + f'(x)) + \beta(xg(x) + g'(x)) = \alpha T(f(x)) + \beta T(g(x)).
\end{aligned}$$

2.1.14. See book for problem.

Answer: (a) Suppose $T \in L(V, W)$ is $1-1$ and I is linearly independent. Let $J = T(I)$. Suppose that $w_1, \ldots, w_k \in J$ and $\alpha_1 w_1 + \ldots + \alpha_k w_k = 0$. Each $w_i = Tv_i$ for some $v_i \in I$ so we see that $T(\alpha_1 v_1 + \ldots + \alpha_k v_k) = 0$, meaning $\alpha_1 v_1 + \ldots + \alpha_k v_k \in N(T) = \{0\}$, i.e., $\alpha_1 v_1 + \ldots + \alpha_k v_k = 0$. Hence, since I is linearly independent, $\alpha_1 = \ldots = \alpha_k = 0$ and so J is linearly independent.

On the other hand, if T takes linearly independent sets to linearly independent sets, we cannot have $Tv = 0_W$ for $v \neq 0_V$ (because $\{v\}$ is linearly independent and $\{0_W\}$ is not). Hence, $N(T) = \{0_V\}$ and T is $1-1$.

(b) Suppose $T \in L(V, W)$ is $1-1$ and $S \subseteq V$. If S is linearly independent, then so is $T(S)$, by (a). Conversely, if $T(S)$ is linearly independent, then so is every subset $T(A)$ of $T(S)$ where $A \subseteq S$ is finite, and hence so is every such A by problem 2.1.13 (using the fact that T is $1-1$, so T maps the elements of $A = \{v_1, v_2, \ldots, v_k\}$ to distinct elements of $T(A) = \{w_1, w_2, \ldots, w_k\}$ as in the setup for 2.1.13). But if every finite subset A of S is linearly independent, so is all of S.

(c) Since β is a spanning set for V, $T(\beta)$ is a spanning set for $R(T)$, and $R(T) = W$ since T is onto. By (a) or (b), since T is $1-1$ $T(\beta)$ is linearly independent. So it is a basis for W.

2.1.17. See book for problem.

Answer: It is easier to prove the contrapositive in both cases. (a) If T is onto then $\dim W = \text{rank}\, T \leq \text{rank}\, T + \text{null}\, T = \dim V$. (b) If T is $1-1$ then $\text{null}\, T = 0$ so that $\dim V = \text{rank}\, T + \text{null}\, T = \text{rank}\, T \leq \dim W$, where the last inequality follows because $R(T)$ is a subspace of W, so $\text{rank}\, T = \dim R(T) \leq \dim W$.

2.1.18. See book for problem.

Answer: If $N(T) = R(T)$ then $\text{rank}\, T = \text{null}\, T$ and so $\text{rank}\, T = \text{null}\, T = \dim V/2$ by the Rank-Nullity Theorem. That means for $T \in L(\mathbf{R}^2 \to \mathbf{R}^2)$ we should look for something with range and nullspace both of dimension 1. For example, $T(x_1, x_2) = (x_2, 0)$ has $N(T) = \{(x_1, 0) \mid x_1 \in \mathbf{R}\}$ and $R(T) = \{(x_2, 0) \mid x_2 \in \mathbf{R}\}$ which are the same set, the line where the second coordinate is 0.

2.4.1. See book for problem. Answers in book.

2.4.2. See book for problem. Answers in book.

2.4.14. See book for problem.

Answer: Obviously one solution is $T\begin{bmatrix} a & a+b \\ 0 & c \end{bmatrix} = (a,b,c)$. One problem with this is that it assumes we are given the elements of V in the form of $\begin{bmatrix} a & a+b \\ 0 & c \end{bmatrix}$, whereas what we are likely to see is just $\begin{bmatrix} a & d \\ 0 & c \end{bmatrix}$ (since $d = a+b$ can be any number). Then $T\begin{bmatrix} a & d \\ 0 & c \end{bmatrix} = (a, d-a, c)$ would agree with our previous definition. But we could alternatively take $T'\begin{bmatrix} a & d \\ 0 & c \end{bmatrix} = (a,d,c)$.

2.4.15. See book for problem.

Answer: Suppose T is an isomorphism. Since β spans V, $T(\beta)$ spans $R(T)$ and $R(T) = W$ since T is onto. Also, since T is $1-1$, it preserves the size of sets, so $|T(\beta)| = |\beta| = n$. Hence $T(\beta)$ is a basis of W.

Conversely, suppose $T(\beta)$ is a basis for W. Since β spans V, $T(\beta)$ spans $R(T)$, so we must have $R(T) = W$, which means T is onto, and hence rank $T = \dim W = n$. But then null $T = \dim V - \text{rank } T = n - n = 0$, so T is also $1-1$. So T is an isomorphism.

2.4.17. See book for problem.

Answer: (a) Let T_0 be the restriction of T to V_0. It is still linear, so $T(V_0) = R(T_0)$ is a subspace of W.
(b) Since $N(T) = \{0\}$, $N(T_0) = \{0\}$ as well (T_0 cannot send more things to 0 than T does). So null $T_0 = 0$ so $\dim T(V_0) = \dim R(T_0) = \text{rank } T_0 = \dim V_0 - \text{null } T_0 = \dim V_0$.

2.3.1. See book for problem. Answers in book.

2.3.2. See book for problem. Incomplete answers in book.

2.3.13. See book for problem.

Answer: $\text{tr }(AB) = \sum_{i=1}^{n}(AB)_{ii} = \sum_{i=1}^{n}(\sum_{k=1}^{n} A_{ik}B_{ki}) = \sum_{k=1}^{n}(\sum_{i=1}^{n} A_{ik}B_{ki})$ (changing the order of summation) $= \sum_{k=1}^{n}(\sum_{i=1}^{n} B_{ki}A_{ik}) = \sum_{k=1}^{n}(BA)_{kk} = \text{tr }(BA)$. For $\text{tr } A = \text{tr } A^{\mathrm{T}}$, just observe that $A_{ii} = (A^{\mathrm{T}})_{ii}$.

2.4.6. See book for problem.

Answer: If $AB = 0$ then $B = IB = (A^{-1}A)B = A^{-1}(AB) = A^{-1}0 = 0$.

2.4.16. See book for problem.

X7. Describe the action of the linear transformations L_A for the following matrices. For example, if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ you should say: we have $L_A \in L(\mathbf{R}^3, \mathbf{R}^2)$ with $L_A(x_1,x_2,x_3) = (x_1+2x_2+3x_3, 4x_1+5x_2+6x_3)$.

(a) $A = \begin{bmatrix} 2 & 4 \\ -1 & -7 \end{bmatrix}$. (b) $B = [\,3 \quad -2 \quad 7 \quad 9\,]$. (c) $C = \begin{bmatrix} 1 & 0 \\ -5 & 2 \\ 0 & 1 \\ 17 & -1 \end{bmatrix}$.

Answer: (a) $L_A \in L(\mathbf{R}^2, \mathbf{R}^2)$ by $L_A(x_1,x_2) = (2x_1 + 4x_2, -x_1 - 7x_2)$.
(b) $L_B \in L(\mathbf{R}^4, \mathbf{R})$ by $L_B(x_1,x_2,x_3,x_4) = 3x_1 - 2x_2 + 7x_3 + 9x_4$.
(c) $L_C \in L(\mathbf{R}^2, \mathbf{R}^4)$ by $L_C(x_1,x_2) = (x_1, -5x_1 + 2x_2, x_2, 17x_1 - x_2)$.

X8. A number of geometric transformations in $\mathbf{R}^2$ or $\mathbf{R}^3$ are known to be linear transformations. By using their effect on the standard basis vectors, find the standard matrices of the following linear transformations.

(a) $F_1 : \mathbf{R}^2 \to \mathbf{R}^2$ which reflects points in the x-axis (line $y = 0$).
(b) $F_2 : \mathbf{R}^2 \to \mathbf{R}^2$ which reflects points in the line $y = -x$.
(c) $R_{2\pi/3} : \mathbf{R}^2 \to \mathbf{R}^2$ which rotates points $2\pi/3$ around the origin (angles measured anticlockwise).
(d) $S_3 : \mathbf{R}^2 \to \mathbf{R}^2$ which shears points parallel to the y-axis, so that every point (x, y) is moved $3x$ units upwards.
(e) $F_3 : \mathbf{R}^3 \to \mathbf{R}^3$ which reflects points in the plane $y = 0$.
(f) $P : \mathbf{R}^3 \to \mathbf{R}^3$ which rotates points around the x axis by 90° clockwise (as we look back from infinity in the x direction).
(g) $T : \mathbf{R}^3 \to \mathbf{R}^3$ which stretches vectors by factors of 2 in the x direction, 3 in the y direction, and 5 in the z direction (so that $(1, 1, 1)$, for example, maps to $(2, 3, 5)$).

Answer: (a) $[F_1] = [F_1(1,0) \quad F_1(0,1)] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. (b) $[F_2] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.
(c) $[R_{2\pi/3}] = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$. (d) $[S_3] = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$. (e) $[F_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
(f) $[P] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$. (g) $[T] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

X9. Find the standard matrices of the following linear transformations.

(a) $T : \mathbf{R}^3 \to \mathbf{R}^4$ by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, 4x_1 + 7x_3, 9x_2, -6x_1 + x_2 - x_3)$.
(b) $S : \mathbf{R}^5 \to \mathbf{R}^2$ by $S(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_3 + x_5, x_2 + x_4)$.
(c) $R : \mathbf{R}^3 \to \mathbf{R}^3$ by $R(a, b, c) = (9c - a, 7b - c, 2b - 3a)$.

Answer: (a) $[T] = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 7 \\ -0 & 9 & 0 \\ -6 & 1 & -3 \end{bmatrix}$. (b) $[S] = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$. (c) $[R] = \begin{bmatrix} -1 & 0 & 9 \\ 0 & 7 & -1 \\ -3 & 2 & 0 \end{bmatrix}$.

X10. In each case you are given the coordinate vector $[v]_B$ of a vector v relative to the ordered basis B of vector space V; find v.

(a) $[v]_B = (7, 6, 5, 4, 3, 2)$, $B = \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$ in $\mathbf{R}^{3\times 2}$.
(b) $[v]_B = (5, 2, 1, 7)$, $B = (x^3, 1, x, x^2)$ in $P_3(\mathbf{R})$.
(c) $[v]_B = (3, 1, 4, 8)$, $B = (\sin, \cos, \exp, \log)$ in the subspace $V = \operatorname{span} B$ of $C((0, \infty))$, continuous real functions defined on the positive numbers.
(d) $[v]_B = (5, 6, 7, 8)$, B is the reordering (e_3, e_2, e_4, e_1) of the standard basis of $\mathbf{R}^4$.
(e) $[v]_B = (9, 8, 6, 4)$, $B = \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right)$ in $\mathbf{R}^{2\times 2}$.

Answer: (a) $v = \begin{bmatrix} 7 & 6 \\ 5 & 4 \\ 3 & 2 \end{bmatrix}$. (b) $v = 5x^3 + 2 + x + 7x^2 = 2 + x + 7x^2 + 5x^3$.
(c) $v = 3\sin + \cos + 4\exp + 8\log$. (d) $v = 5e_3 + 6e_2 + 7e_4 + 8e_1 = (8, 6, 5, 7)$. (e) $v = \begin{bmatrix} 27 & 23 \\ 17 & 9 \end{bmatrix}$.

X11. In each case find the coordinate vector $[v]_B$ of the given vector v relative to the ordered basis B of vector space V.

(a) $v = (1, 2, 3, 4)$, B is the standard basis of $V = \mathbf{R}^4$.
(b) $v = (5, 6, 7, 8)$, B is the reordering (e_3, e_2, e_4, e_1) of the standard basis of $V = \mathbf{R}^4$.
(c) $v = 2 + x^3$, B is the standard basis $(1, x, x^2, \ldots, x^6)$ of $V = P_6(\mathbf{R})$.
(d) $v = 3 + 4x + x^4$, B is the ordered basis $(1, x - 1, x^2 - x, x^3 - x^2, x^4 - x^3)$ of $V = P_4(\mathbf{R})$.
(e) $v = \begin{bmatrix} 2 & 1 \\ -3 & 7 \end{bmatrix}$, B is the ordered basis $\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right)$ of $V = \mathbf{R}^{2\times 2}$.

Answer: (a) $[v]_B = (1, 2, 3, 4)$. (b) $[v]_B = (7, 6, 8, 5)$. (c) $[v]_B = (2, 0, 0, 1, 0, 0, 0)$.
(d) $[v]_B = (8, 5, 1, 1, 1)$. (e) $[v]_B = (7, -10, 4, 1)$.

2.2.3. See book for problem. Answers in book.

2.2.4. See book for problem.

Answer: Write $\beta = (E_1, E_2, E_3, E_4)$, then

$$[T]_\beta^\gamma = [[TE_1]_\gamma \; [TE_2]_\gamma \; [TE_3]_\gamma \; [TE_4]_\gamma] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

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2.3.3. See book for problem. Answers in book.

2.4.2. See book for problem. Answers in book.

2.4.20. See book for problem.

Answer: We know that $L_A\phi_\beta = \phi_\gamma T$. Suppose $v \in V$ and $x \in F^n$ satisfy $\phi_B v = x$. Then

$Tv = 0 \Leftrightarrow \phi_\gamma(Tv) = 0$ because ϕ_γ is $1 - 1$

$\Leftrightarrow L_A(\phi_\beta v) = L_A x = 0$.

So if $x \in \phi_B(N(T))$ then $x = \phi_B v$ where $Tv = 0$, so $L_A x = 0$ and $x \in N(L_A)$. Conversely, if $x \in N(L_A)$ then take $v = \phi_\beta^{-1} x$, so that $x = \phi_\beta v$. Then $L_A x = 0$, so $Tv = 0$, $v \in N(T)$, and $x = \phi_B v \in \phi_B(N(T))$. Thus, $\phi_B(N(T)) = N(L_A)$.

By 2.4.17(b), since ϕ_β is an isomorphism, $\dim N(T) = \dim \phi_B(N(T)) = \dim N(L_A)$ and hence null $T =$ null L_A. Then by the Rank-Nullity Theorem, rank $T = n -$ null $T = n -$ null $L_A =$ rank L_A.

2.5.1. See book for problem. Answers in book.

2.5.2. See book for problem. Incomplete answers in book.

2.5.3. See book for problem. Incomplete answers in book.

2.5.5. See book for problem. Answers in book.

2.5.10. See book for problem.

Answer: We know that $\mathrm{tr}\, XY = \mathrm{tr}\, YX$. Suppose A and B are similar, so $B = Q^{-1}AQ$ for some Q. Then $\mathrm{tr}\, B = \mathrm{tr}\, (Q^{-1}A)Q = \mathrm{tr}\, Q(Q^{-1}A) = \mathrm{tr}\, QQ^{-1}A = \mathrm{tr}\, IA = \mathrm{tr}\, A$.

2.5.11. See book for problem.

Answer: (a) We know in general that $[TS]_\alpha^\gamma = [T]_\beta^\gamma [S]_\alpha^\beta$. Here we take $S = T = I_V$ and we get $[I_V]_\alpha^\gamma = [I_V I_V]_\alpha^\gamma = [I_V]_\beta^\gamma [I_V]_\alpha^\beta = RQ$, as required.

(b) We know in general that $[T^{-1}]_\beta^\alpha = ([T]_\alpha^\beta)^{-1}$. Let $T = I_V$, then $[I_V]_\beta^\alpha = [I_V^{-1}]_\beta^\alpha = ([I_v]_\alpha^\beta)^{-1} = Q^{-1}$, as required.