## Math 2600/5600 - Linear Algebra - Fall 2015

### Extra Problems for Chapter 2

**X7.** Describe the action of the linear transformations $L_A$ for the following matrices. For example, if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ you should say: we have $L_A \in L(\mathbf{R}^3, \mathbf{R}^2)$ with $L_A(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, 4x_1 + 5x_2 + 6x_3)$.

(a) $A = \begin{bmatrix} 2 & 4 \\ -1 & -7 \end{bmatrix}$. (b) $B = [\,3 \quad -2 \quad 7 \quad 9\,]$. (c) $C = \begin{bmatrix} 1 & 0 \\ -5 & 2 \\ 0 & 1 \\ 17 & -1 \end{bmatrix}$.

**X8.** A number of geometric transformations in $\mathbf{R}^2$ or $\mathbf{R}^3$ are known to be linear transformations. By using their effect on the standard basis vectors, find the standard matrices of the following linear transformations.

(a) $F_1 : \mathbf{R}^2 \to \mathbf{R}^2$ which reflects points in the $x$-axis (line $y = 0$).
(b) $F_2 : \mathbf{R}^2 \to \mathbf{R}^2$ which reflects points in the line $y = -x$.
(c) $R_{2\pi/3} : \mathbf{R}^2 \to \mathbf{R}^2$ which rotates points $2\pi/3$ around the origin (angles measured anticlockwise).
(d) $S_3 : \mathbf{R}^2 \to \mathbf{R}^2$ which shears points parallel to the $y$-axis, so that every point $(x, y)$ is moved $3x$ units upwards.
(e) $F_3 : \mathbf{R}^3 \to \mathbf{R}^3$ which reflects points in the plane $y = 0$.
(f) $P : \mathbf{R}^3 \to \mathbf{R}^3$ which rotates points around the $x$ axis by $90°$ clockwise (as we look back from infinity in the $x$ direction).
(g) $T : \mathbf{R}^3 \to \mathbf{R}^3$ which stretches vectors by factors of 2 in the $x$ direction, 3 in the $y$ direction, and 5 in the $z$ direction (so that $(1, 1, 1)$, for example, maps to $(2, 3, 5)$).

**X9.** Find the standard matrices of the following linear transformations.

(a) $T : \mathbf{R}^3 \to \mathbf{R}^4$ by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, 4x_1 + 7x_3, 9x_2, -6x_1 + x_2 - x_3)$.
(b) $S : \mathbf{R}^5 \to \mathbf{R}^2$ by $S(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_3 + x_5, x_2 + x_4)$.
(c) $R : \mathbf{R}^3 \to \mathbf{R}^3$ by $R(a, b, c) = (9c - a, 7b - c, 2b - 3a)$.

**X10.** In each case you are given the coordinate vector $[v]_B$ of a vector $v$ relative to the ordered basis $B$ of vector space $V$; find $v$.

(a) $[v]_B = (7, 6, 5, 4, 3, 2)$, $B = \left( \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$ in $\mathbf{R}^{3\times 2}$.
(b) $[v]_B = (5, 2, 1, 7)$, $B = (x^3, 1, x, x^2)$ in $P_3(\mathbf{R})$.
(c) $[v]_B = (3, 1, 4, 8)$, $B = (\sin, \cos, \exp, \log)$ in the subspace $V = \operatorname{span} B$ of $C((0, \infty))$, continuous real functions defined on the positive numbers.
(d) $[v]_B = (5, 6, 7, 8)$, $B$ is the reordering $(e_3, e_2, e_4, e_1)$ of the standard basis of $\mathbf{R}^4$.
(e) $[v]_B = (9, 8, 6, 4)$, $B = \left( \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right)$ in $\mathbf{R}^{2\times 2}$.

**X11.** In each case find the coordinate vector $[v]_B$ of the given vector $v$ relative to the ordered basis $B$ of vector space $V$.

(a) $v = (1, 2, 3, 4)$, $B$ is the standard basis of $V = \mathbf{R}^4$.
(b) $v = (5, 6, 7, 8)$, $B$ is the reordering $(e_3, e_2, e_4, e_1)$ of the standard basis of $V = \mathbf{R}^4$.
(c) $v = 2 + x^3$, $B$ is the standard basis $(1, x, x^2, \dots, x^6)$ of $V = P_6(\mathbf{R})$.
(d) $v = 3 + 4x + x^4$, $B$ is the ordered basis $(1, x - 1, x^2 - x, x^3 - x^2, x^4 - x^3)$ of $V = P_4(\mathbf{R})$.
(e) $v = \begin{bmatrix} 2 & 1 \\ -3 & 7 \end{bmatrix}$, $B$ is the ordered basis $\left( \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right)$ of $V = \mathbf{R}^{2\times 2}$.