**Math 2600/5600 - Linear Algebra - Fall 2015**

**Review Sheet for Test 2**

Test 2 covers material from the Rank-Nullity Theorem up to and including the theory of systems of linear equations (but not including checking the rank of the augmented matrix). Earlier material may be included if it is necessary as background.

Below I try to give an indication of various things from the course that you are expected to know, and the level at which you need to know them. This list DOES NOT include everything that you are supposed to know.

Remember that anything that is on a homework or practice problem is fair game for the test. One of the best ways to study for the test is to make sure you can do all the practice problems.

- Definitions that you should know and be able to state:

*one-to-one (1-1)* or *injective*;
*onto* or *surjective*;
*bijection* or *1-1 correspondence*;
*invertible* linear transformation or *(vector space) isomorphism*;
*isomorphic* vector spaces;
*reflexive*, *symmetric*, *transitive* relations;
*equivalence relation*;
*$m \times n$ matrix*;
*transpose* $A^{\mathrm{T}}$;
*dot product*;
*matrix multiplicatinn*;
*$n \times n$ identity matrix*;
*inverse matrix* $A^{-1}$;
*column space* colsp $A$;
*row space* rowsp $A$;
$[T]$ = *standard matrix of* $T$;
linear transformation $L_A$ for matrix $A$;
$R(A)$, $N(A)$, rank $A$, null $A$ for matrix $A$;
$[v]_B$ = *coordinate vector of $v$ relative to $B$*;
$[T]_B^C$ = *matrix of $T$ relative to $B$ and $C$*;
$[T]_B = [T]_B^B$ = *matrix of $T$ relative to $B$*;
$[I]_B^C$ = *change of coordinate matrix from $B$ to $C$*;
*similar* square matrices;
*elementary operations* for system of linear equations;
*equivalent* systems of linear equations;
*elementary row operations*;
*elementary column operations*;
*elementary matrix*;
*(full) pivot* on a matrix element;
*row echelon form* matrix;
*reduced row echelon form* matrix;
*solution* and *solution set*;
*coefficient matrix* of a system;
*consistent* and *inconsistent* systems;
*homogeneous* and *inhomogeneous* systems;

Some of these definitions were given differently in class from the version in the book, or were not given in the book at all. You should know the version from class:

A relation $\sim$ is *reflexive* if $x \sim x$ for every $x$; *symmetric* if $x \sim y \Rightarrow y \sim x$; and *transitive* if $x \sim y$ and $y \sim z \Rightarrow x \sim z$. An *equivalence relation* is a relation that is reflexive, symmetric and transitive.

For a matrix $A \in F^{m\times n}$, its *range* $R(A)$, *nullspace* $N(A)$, *rank* rank $A$ and *nullity* null $A$ are the range, nullspace, rank and nullity (respectively) of the associated linear transformation $L_A \in L(F^n, F^m)$.

For $A \in F^{m\times n}$ its *column space* colsp $A$ is the subspace of $F^m$ spanned by the columns of $A$, and its *row space* rowsp $A$ is the subspace of $F^n$ spanned by the rows of $A$.

The *dot product* of $a = (a_1, a_2, \dots, a_n), b = (b_1, b_2, \dots, b_n) \in F^n$ is $a \cdot b = a_1b_1 + a_2b_2 + \dots + a_nb_n$.

Matrix multiplication is defined as follows: For $A \in F^{m\times n}$ and $B \in F^{n\times p}$, $AB$ is the matrix in $F^{m\times p}$ such that $(AB)_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}$ for all $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, p$.

An *elementary matrix* is defined to be a matrix obtained by applying an elementary row operation to an identity matrix. (Definition does not include elementary column operations, unlike book.)

The *standard matrix* of $T \in L(F^n, F^m)$ is the $m \times n$ matrix $[T] = [\ Te_1 \mid Te_2 \mid \ \ldots \ \mid Te_n\ ]$ where $(e_1, e_2, \ldots, e_n)$ is the standard basis of $F^n$. It has the property that $T(v) = [T]v$.

A *(full) pivot* on *pivot entry* $A_{ij} \neq 0$
(1) makes $A_{ij} = 1$ by dividing row $i$ by $A_{ij}$, then
(2) zeroes out column $j$: makes $A_{kj} = 0$ for $k \neq i$ by adding multiples of row $i$ to other rows.

The *reduced row echelon form* of a matrix has
(1) all zero rows at bottom,
(2) first nonzero entry of row further to right than previous row,
(3) first nonzero entry of row is a 1, and
(4) no other nonzero entries in column of first nonzero entry of a row.
*Row echelon form* means just (1) and (2).

- There are certain things that you should be able to do:

Do various coordinate conversions and compute matrices of a linear transformation with respect to different bases.

Reduce a matrix to the form $\left[\begin{array}{c|c} I & 0 \\ \hline 0 & 0 \end{array}\right]$ using elementary row and column operations.

Reduce a matrix to reduced row echelon form using the Gauss-Jordan algorithm (see below).

Find the rank of a matrix by reducing it to a simple form of some kind using elementary row and/or column operations (either of the two algorithms immediately above can be used).

Find the inverse of a matrix, or determine that it is noninvertible, by applying the Gauss-Jordan algorithm to an augmented matrix.

Use a matrix to determine the rank of a linear transformation, or determine whether a linear transformation is invertible.

**Gauss-Jordan Elimination Algorithm:** Puts matrix into reduced row echelon form by sequence of pivots. Call a row *dead* if have pivoted in it; *live* otherwise.

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repeat
    choose pivot column: leftmost column with nonzero entry in a live row
    choose pivot row: live row with nonzero entry in pivot column
    make pivot row first live row by exchanging if necessary
    do (full) pivot on pivot entry
until all rows are dead or zero
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- There are certain theorems you should know by name, and be able to state and use:

Rank-Nullity Theorem
Finite Dimensional Vector Spaces are Boring Theorem

- There are also certain formulae you should know:

$[T]_B^C = [\ [Tx_1]_C \mid [Tx_2]_C \mid \ \ldots \ \mid [Tx_n]_C\ ]$ where $B = (x_1, x_2, \ldots, x_n)$
$[Tv]_C = [T]_B^C [v]_B$
$[UT]_B^D = [U]_C^D [T]_B^C$
$[T^{-1}]_C^B = ([T]_B^C)^{-1}$, assuming $T$ is invertible