Math 2600/5600 - Linear Algebra - Fall 2015

Review Sheet for Test 1

Test 1 covers material up to and including the definition of range and nullspace of a linear transformation, and ways of calculating these.

Below I try to give an indication of various things from the course that you are expected to know, and the level at which you need to know them. This list DOES NOT include everything that you are supposed to know.

Remember that anything that is on a homework or practice problem is fair game for the test. One of the best ways to study for the test is to make sure you can do all the practice problems.

- Definitions that you should know and be able to state:

field;
specific fields $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{C}$, $\mathbf{Z}_p$;
vector space or *linear space*;
abelian group;
vector space axioms;
row vectors, in $F^{1\times n}$;
column vectors, in $F^{n\times 1}$;
set of vectors F^n;
set of $m \times n$ matrices $M_{m\times n}(F)$ or $F^{m\times n}$;
set of polynomial expressions with coefficients in F, $P(F)$;
set of polynomial expressions of degree at most n, $P_n(F)$;
set of functions $\mathcal{F}(X, Y) = Y^X$;
subspace of a vector space;
transpose of a matrix;
symmetric matrix;
trace of a matrix;
symmetric difference of two sets;
linear combination;
span of S, span S;
spanning set for V;
nontrivial linear combination;
linearly dependent and *linearly independent*;
basis for a vector space;
finite-dimensional and *infinite-dimensional*;
standard basis for F^n;
dimension of V, dim V;
linear transformation from V to W, *linear*, or *vector space homomorphism*;
$L(V, W)$;
identity linear transformation $I = I_V$;
zero linear transformation 0;
linear operator on V;
nullspace or kernel, $N(T)$;
range or image, $R(T)$;
rank of T, rank T;
nullity of T, null T.

Some of these definitions were given differently in class from the version in the book. You should know the version from class:

Definition of a vector space, from the Vector Spaces/Fields handout. (You should know all the axioms; the exact order and numbering does not matter.)

Definition of a field, from the Vector Spaces/Fields handout. (You should know all the axioms; the exact order and numbering does not matter.)

Definition of linear combination and span: Let V be a vector space over F, with a finite collection of vectors $v_1, v_2, \ldots, v_k \in V$ and with $\alpha_1, \alpha_2, \ldots, \alpha_k \in F$. The expression

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_k v_k$$

is called a *linear combination* of $v_1, v_2, \ldots, v_k$. The zero vector 0 is considered to be a linear combination of an empty collection of vectors. For any $S \subseteq V$, a *linear combination of elements of S* is a linear combination of finitely many (possible zero) vectors from S. The *span* of S, span S, is the set of all linear combinations of elements of S. Note that span $\emptyset = \{0\}$ (set with just zero vector).

Definition of linear dependence and independence: A collection of vectors in a vector space is *linearly dependent* if it contains finitely many vectors $v_1, v_2, \ldots, v_k$ $(k \geq 1)$ and scalars $\alpha_1, \alpha_2, \ldots, \alpha_k$ **not all zero** such that $\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_k v_k = 0$. In other words, there is a *nontrivial linear combination* of $v_1, v_2, \ldots, v_k$ that is 0. Otherwise the collection is *linearly independent*. (Only *trivial linear combinations*, empty or all coefficients zero, give 0.)

- There are certain things that you should be able to do:

- Prove something is a subspace using the Subspace Theorem (or streamlined version).

- Prove something is a linear transformation, using (LT1) and (LT2), or using (LTP).

- Prove something is a subfield, using the Subfield Theorem.

At this point, we can only do fairly simple calculations of the following kinds. Later we will develop tools to handle more complicated cases:

- Determine where a collection of vectors is linearly independent.

- Determine whether one vector belongs to the span of a finite set of vectors.

- Describe range and nullspace of a linear transformation.

- Reduce a spanning set to a basis.

- Use a spanning set to extend an independent set to a basis.

- There are certain theorems you should know by name, and be able to state and use:

Subspace Theorem (original or streamlined, either is OK);
Subfield Theorem (**corrected** version from handout);
Representation Theorem (as in class, or Theorem 1.8 in book);
Replacement Theorem (see below).

Theorem B4 (Replacement Theorem): Let $I, S \subseteq V$, where I is lin. ind. and S is a *finite* spanning set for V. Then $|I| \leq |S|$ (so $|I|$ is finite) and there is $S' \subseteq S$ with $|S'| = |I|$ such that $(S - S') \cup I$ is a spanning set for V.