Math 2600/5600 - Linear Algebra - Fall 2015

Review Sheet for Final Examination

The final examination is cumulative, covering all material we have seen this semester. This sheet covers only material that is not already on one of the earlier review sheets for Tests 1, 2 and 3. You are also expected to know everything from the earlier review sheets.

Below I try to give an indication of various things from the course that you are expected to know, and the level at which you need to know them. This list DOES NOT include everything that you are supposed to know.

Remember that anything that is on a homework or practice problem is fair game for the final examination. One of the best ways to study for the test is to make sure you can do all the practice problems.

There are three areas that were not covered on earlier test review sheets.

1. Solutions of linear equations. Some material from Chapter 3 was not covered on Test 2, and was not explicitly covered on Test 3, although it was covered implicitly (to find an eigenspace you have to solve a system of linear equations).

- You should know and be able to state definitions of the following.

augmented matrix of a system of linear equations;

leading entry and *leading column* of a row echelon form matrix;

leading variable in a system of linear equations whose augmented matrix is in row echelon form;

lower and *upper* pivots on a matrix element.

The following definitions from above were not in the book:

For a row echelon matrix, a *leading entry* is the first nonzero entry in a given row, and a *leading column* is a column containing a leading entry. A *leading variable* in a system of linear equations is a variable corresponding to a leading column in the augmented matrix.

A *lower pivot* is like a full pivot but changes only the pivot entry and entries below it. An *upper pivot* is like a full pivot but changes only the pivot entry and entries above it.

- There are certain things that you should be able to do:

Reduce a matrix to reduced row echelon form using the Gaussian elimination algorithm (see below) (as well as using Gauss-Jordan algorithm).

Gaussian Elimination Algorithm: Puts matrix into reduced row echelon form by sequence of lower and upper pivots.

Forward pass: like Gauss-Jordan elimination, but do lower pivot on each entry instead of full pivot.

Backward pass: go through pivot entries in reverse order, doing upper pivot on each one.

Solve a system of linear equations by reducing its augmented matrix to reduced row echelon form (by either Gauss-Jordan or Gaussian elimination), then solving for leading variables.

Find a basis for the solution set of a homogeneous system of linear equations.

Find a solution set using a particular solution and the solution set of the corresponding homogeneous system (forgot to mention this for Test 2!).

2. Cayley-Hamilton Theorem. We just covered the statement and applications.

- There is a theorem you should know by name, and be able to state and use:

Cayley-Hamilton Theorem

- There are certain things that you should be able to do:

Use the Cayley-Hamilton Theorem to find the inverse of a matrix or linear transformation in terms of its powers.

Use the Cayley-Hamilton Theorem to express large powers of a matrix or linear transformation in terms of smaller powers.

3. Inner product spaces. We covered Sections 6.1 and 6.2 but only for real inner product spaces. You do not have to know any definitions or results for complex inner product spaces.

Since we are only covering real inner product spaces, you may be a bit confused reading the textbook. Therefore, lecture notes on inner product spaces are available from the main class web page and you should use the definitions from the notes, not the book.

- You should know and be able to state definitions of the following **from the 'Lecture notes on inner product spaces' posted on the class web site**. Do not use the book's definitions.

real inner product space and *real inner product*;
positive definite property of inner product;
bilinear;
length or *magnitude* in terms of inner product;
angle in terms of inner product and length;
unit vector in direction of x;
orthogonal or *perpendicular* vectors, $x \perp y$;
norm (the properties this implies, not just definition of length or magnitude);
triangle inequality;
orthogonal set of vectors;
orthonormal set of vectors;
orthogonal projection of y *onto* $x \neq 0$, $\text{proj}_x y$ (definition AND formula, and make sure you know which is which!);
orthogonal complement $S^\perp$ of set S;
orthogonal projection of y *onto subspace* W, $\text{proj}_W y$ (see note below).

Again, you do **NOT** have to know the definition of a complex inner product space.

The following definition was not stated very clearly in the lecture notes; use the definition here:

Suppose W is a subspace of real inner product space V. For $y \in V$, we say w is the *orthogonal projection of* y *onto* W, $\text{proj}_W y$, if $y = w + z$ with $w \in W$ and $z \in W^\perp$.

- There are certain things that you should be able to do:

Compute a projection $\text{proj}_x y$.

Construct a set of orthogonal vectors using the Gram-Schmidt algorithm.

- There are certain results you should know and be able to state:

Cauchy-Schwartz inequality.

Lemma about unique representation of a vector in terms of nonzero orthogonal vectors, including formula for coordinates of y relative to $(v_1, v_2, \ldots, v_k)$ in terms of inner product: $\alpha_i = \langle y, v_i\rangle / \langle v_i, v_i\rangle$.

Consequence about nonzero orthogonal vectors being linearly independent.

Dimension result and duality result for orthogonal complements: if V finite-dimensional with subspace W, then (a) $\dim W + \dim W^\perp = \dim V$, and (b) $(W^\perp)^\perp = W$.