

Math 2600/5600 - Linear Algebra - Fall 2015
Assignment 3, due in class, Friday, 23rd October

Remember:

- Solutions to problems should be fully explained, using clear English sentences where necessary.
- Matrix calculations may be done using **LA**, or some other computational tool. However, your solutions should show details of your calculations, not just the final answers. Show intermediate steps in your calculations, including any matrix inverses you compute. If answers can be given exactly as fractions, do so; do not give decimal approximations.
- A short tutorial on using **LA** is attached to this assignment handout.
- Solutions should be written (or typed) neatly on one side only of clean paper with straight (not ragged) edges.
- Multiple pages should be stapled (not clipped or folded) together.

A3.1. Suppose we are working with the ordered basis

$$A = (x^4 + x^3 + x^2 + x + 1, x^3 + x^2 + x + 1, x^2 + x + 1, x + 1, 1)$$

of $P_4(\mathbf{R})$.

(a) If $[p(x)]_A = (5, -3, 0, 4, 0)$, what is $p(x)$?

(b) What is $[1 - 7x + 5x^2 - 2x^3 + 4x^4]_A$? (You can do this without using any matrices.)

A3.2. Find the standard matrix of the linear transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^4$ by

$$T(a, b, c) = (a + b, c - 2a, 5c - b - a, b + 7c).$$

A3.3. Find the matrix $[A]_C^B$ of the linear transformation $A : P_3(\mathbf{R}) \rightarrow \mathbf{R}^4$ by

$$A(p(x)) = (p(0), p'(1), p''(2), p'''(3)) \quad (p', p'', p''' \text{ indicate first, second, third derivatives of } p)$$

where $C = (1, 1 + x, x + x^2, x^2 + x^3)$ and B is the standard ordered basis in \mathbf{R}^4 .

A3.4. One application of alternative coordinate systems is that sometimes it is easy to compute the matrix of a linear transformation in a special coordinate system; then we can find the standard matrix using our conversion formulae.

Consider the linear operator R_π on \mathbf{R}^3 which reflects points in the plane π with equation $2x + 3y + 4z = 0$.

(a) Find the matrix $[R_\pi]_C$ with respect to the ordered basis $C = ((2, 3, 4), (3, -2, 0), (2, 0, -1))$. (From the way we set up the equation of a plane, we know the first vector in C is normal to π . Show that the other two vectors in C lie in π . Then use these facts to find the matrix.)

(b) Use your answer to (a) to find the standard matrix of R_π .

(c) What is $R_\pi(7, 1, 2)$?

(d) Give a general formula for $R_\pi(x, y, z)$.

A3.5. Find a single 3×3 (real) matrix F so that left-multiplying a $3 \times n$ matrix A by F , forming FA , gives the same result as applying all four of the following elementary row operations, in the given order, to A .

- (1) Add 2 times row 1 to row 3.
- (2) Exchange rows 1 and 2.
- (3) Add 4 times row 1 to row 3.
- (4) Exchange rows 2 and 3.

A short tutorial on LA

First, let's read a matrix into LA. We use `a` to denote the matrix A . If we want a 3×3 matrix A we use the `r` (read) command to enter it into LA. Note that the parts you type appear underlined.

```
LA 1.01/32-bit/rational (Mark Ellingham, 24th August 2015)
Enter h for help at any time
LA >> r 3 3 a
Enter matrix:
1 2 3 1 1 1 1 0 1
a:
      1      2      3
      1      1      1
      1      0      1
```

Now perhaps we want to invert A and compute $A^{-1}v$ for some vector v . We can invert with the `i` command, and multiply two matrices, or a matrix and a vector, with the `mm` (multiply matrix) command:

```
LA >> i a ai
ai:
     -1/2      1      1/2
         0      1      -1
         1/2     -1      1/2
LA >> r 3 1 v
Enter matrix:
4 5 6
v:
      4
      5
      6
LA >> mm ai v aiv
aiv:
      6
     -1
      0
```

We can add v and $A^{-1}v$ together, using the `am` (add matrix) command to get u :

```
LA >> am v aiv u
u:
     10
      4
      6
```

Actually the `am` command can form more general linear combinations, like $3v - 2A^{-1}v$:

```
LA >> am 3 v -2 aiv u
u:
      0
     17
     18
```

Or maybe we want to read in another matrix Q and compute $Q^{-1}AQ$. Note that we can only multiply two matrices at a time, so we need to store a partial result in a temporary variable `t`.

```

LA >> r 3 3 q
Enter matrix:
1 0 2
0 1 0
4 0 1
q:
      1      0      2
      0      1      0
      4      0      1
LA >> i q qi
qi:
     -1/7      0      2/7
      0      1      0
      4/7      0     -1/7
LA >> mm qi a t
t:
      1/7     -2/7     -1/7
      1      1      1
      3/7     8/7     11/7
LA >> mm t a qiaq
qiaq:
     -2/7      0      0
      3      3      5
     22/7      2      4

```

LA can also do elementary row and column operations using commands **x** (exchange two rows or two columns), **m** (multiply a row or column by a constant) and **a** (add a multiple of one row/column to another row/column). These commands work on the “current” matrix. For example:

```

LA >> r 3 5 b
Enter matrix:
1 2 3 4 5 0 1 0 1 0 2 1 0 1 2
b:
      1      2      3      4      5
      0      1      0      1      0
      2      1      0      1      2
LA >> x r1 r2      (exchange  $R_1$  and  $R_2$ )
b:
      0      1      0      1      0
      1      2      3      4      5
      2      1      0      1      2
LA >> m c2 7      (multiply  $C_2$  by 7)
b:
      0      7      0      1      0
      1     14      3      4      5
      2      7      0      1      2
LA >> a 3 r1 r2      (add  $3R_1$  to  $R_2$ )
b:
      0      7      0      1      0
      1     35      3      7      5
      2      7      0      1      2

```

Finally, to leave LA use the **q** (quit) command.

```
LA >> q
```