NIL PHENOMENA IN TOPOLOGY (14–15 APRIL 2007)
VANDERBILT UNIVERSITY (NASHVILLE, TN U.S.A.)
PROBLEM SESSION

BRUCE HUGHES, QAYUM KHAN (ORGANIZERS)
IAN HAMBLETON (MODERATOR)

IO. (Ivonne Ortiz, Miami of Ohio)

Does $NK_n(\mathbb{Z}[G], \alpha) = 0$ if and only if $NK_n(\mathbb{Z}[G]) = 0$?

DJP. (Daniel Juan-Pineda, Morelia)

Is $NK_2(\mathbb{Z}[G]) \neq 0$ for some non-trivial finite group G? It is known that $NK_1(\mathbb{Z}[G]) = 0$ if and only if G has p-rank 1. It also is known that $NK_1(\mathbb{Z}[G], \alpha) = 0$ if $C_p \times C_p \not\subset G$.

CH. (Christian Haesemeyer, Illinois)

It seems to be known that $NK_2(\mathbb{Z}[G])$ is non-zero for cyclic groups G and conjectured that it is non-vanishing for all finite groups. Likewise, it is known that $NK_1(\mathbb{F}_p[C_{p^n}])$ does not vanish, and one might guess that $NK_1(\mathbb{F}_p[G]) \neq 0$ for any finite group whose order is divisible by p.

Note that this is suggestive, since $\mathbb{Z}$ has dimension 1 and $\mathbb{F}_p$ has dimension 0. Maybe the following might be true:

Let R be a commutative ring of dimension d and G a finite group such that $R[G]$ is not regular (is that equivalent to "some residue characteristic of R divides the order of G"?). Then $NK_{d+1}(R[G]) \neq 0$, or at least, RG is not K_{d+1}-regular.

Compare to the following conjecture of Vorst:

If R is a finite type algebra over some field, of dimension d, then R is not K_{d+1}-regular. (We recently proved this in the characteristic zero case.)

IH1. (Ian Hambleton, McMaster)

For a finite group G, it is known that $NK_n(\mathbb{Z}[G]) \cong NK_n(\widehat{\mathbb{Z}}_{|G|}[G])$. Suppose G is a finite p-group. How does one study the following relative group?

$$NK_n(\widehat{\mathbb{Z}}_p[G] \longrightarrow \mathbb{F}_p[G])$$

This may have implications in TC-theory.

References: [Dundas] [Hesselholt]

IH2. (Ian Hambleton, McMaster)

Suppose $\mathbb{Z}[G]$ is contained in a maximal order Γ. Consider the cartesian square

$$\begin{array}{ccc} \mathbb{Z}[G] & \longrightarrow & \Gamma \\ \downarrow & & \downarrow \\ \mathbb{Z}[G]/\Gamma & \longrightarrow & \Gamma/n\Gamma. \end{array}$$

Is there a Mayer-Vietoris sequence in NK_n for all $n \geq 1$?
Reference: [Suslin-Voevodsky]

RS. (Ross Staffeldt, New Mexico State)
Consider the Nil-terms associated to fundamental group diagrams

$$G_1 \xleftarrow{i_1} G_0 \xrightarrow{i_2} G_2$$

where i_1 and i_2 are injective.

AR. (Andrew Ranicki, Edinburgh)
K- or L-theory of $G_1 *_{G_0} G_2$ in the non-injective case? The methods of noncommutative localization produce long exact sequences in the injective case, and probably lead to spectral sequences in the general case. Can someone identify a condition that leads to collapsing spectral sequences? Can someone treat, by any ad-hoc methods whatsoever, an example that provides some insight?

BW1. (Bruce Williams, Notre Dame)
Burghelea, Lashof, and Rothenberg have shown, for all i, that

$$\pi_i(C(M \times S^1)) \cong \pi_i(C(M \times I)) \oplus \pi_{i-1}(C \times I) \oplus \pi_i(NC(M))$$

where $NC(M)$ is defined as the homotopy fiber of a map

$$CEmb(M, M \times S^1) \longrightarrow CEmb(M, M \times R^1).$$

Give a direct geometric definition of $NC(M)$.
Reference: [BLR75]

BW2. (Bruce Williams, Notre Dame)
Let $\mathcal{C}(M) := \operatorname{hocolim}_i C(M \times I^i)$ be the stable concordance space. Then (BW1) implies

$$\mathcal{C}(M \times S^1) \simeq \mathcal{C}(M) \times \Omega^{-1}\mathcal{C}(M) \times N\mathcal{C}(M).$$

The algebraic decomposition

$$A(M \times S^1) \simeq A(M) \times \Omega^{-1}A(M) \times N_-A(M) \times N_+A(M)$$

of Klein, Waldhausen, et.al. formally implies that $N\mathcal{C}(M)$ decomposes as a product of two equivalent pieces

$$N_{\pm}C(M) \simeq \Omega N_{\pm}A(M).$$

Give a geometric interpretation of this decomposition.
References: [HKV$^+$01] [HKV$^+$02]

BW3. (Bruce Williams, Notre Dame)
Do we get a decomposition of $NC(M)$ outside the concordance stable range? Does it help that $M = N \times I \times I^j$ for some j?

BW4. (Bruce Williams, Notre Dame)
Algebraic constructions and computations of Hesselholt-Madsen and Grunewald-Klein-Macko yield non-trivial elements in $\pi_0\, N_{\pm}A(M)$. This formally implies the existence of non-trivial elements in $\pi_0\, N_{\pm}\mathcal{C}(M)$ and $\pi_0\, NTOP(M)$. Give geometric descriptions of these elements.

References: [Hesselholt-Madsen] [GKM] [HP00] [Jahren-Rognes-Waldhausen]

JG1. (Joachim Grunewald, Bonn)

Vogel generalizes the class of regular rings and asks the question whether the K-theoretic Nil-groups still vanish for this class of rings. A positive answer to this conjecture would extend to class of groups with trivial Whitehead group.

Reference: [Bih]

JG2. (Joachim Grunewald, Bonn)

Recall that $NA_{\pm}(point) \neq 0$. Is there a notion of "regular" for a space X such that $NA_{\pm}(X) = 0$?

Reference: [GKM]

FC-JG. (Frank Connolly and Joachim Grunewald, Notre Dame and Bonn)

Let G be a finite group. Is $NK_n(\mathbb{Z}[G], \alpha)$ finitely generated over the Verschiebung algebra $\mathcal{V}$? Connolly and da Silva have shown that $NK_0(\mathbb{Z}[G])$ is a finitely generated $\mathcal{V}$-module. Guin-Walery and Loday have shown that $NK_2(\mathbb{Z}[C_p])$ is a cyclic module over $\mathcal{V}/p\mathcal{V}$.

References: [CdS95] [GWL81]

FC1. (Frank Connolly, Notre Dame)

Suppose Γ is a virtual PD group and its Farrell cohomology, $\widehat{H}^*(\Gamma;\mathbb{Z})$, is periodic. Assume Γ has no elements of order 2. Is $\Gamma = \pi_1(M)$ for some closed manifold M such that $\widetilde{M}$ has the homotopy type of a sphere?

FC2. (Frank Connolly, Notre Dame)

$hdim(X)$ = the minimum dimension of a CW complex of homotopy type X. For a crystallographic group Γ of rank n, it is known that $hdim\, B_{vc}(\Gamma) = n + 1$ if $n \geq 2$. Show $hdim(\Gamma) = n$ if Γ is virtually nilpotent of (nilpotent) rank n but is not virtually abelian.

Reference: [CFH]

FC3. (Frank Connolly, Notre Dame)

$UNil_3(\mathbb{Z};\mathbb{Z},\mathbb{Z})$ is an infinite direct sum of cyclic modules over $\mathcal{V} = \mathbb{Z}[V_1, V_2, \ldots]$. Is $UNil_3(\mathbb{Z};\mathbb{Z},\mathbb{Z})$ finitely generated over a slightly larger algebra of operators?

Joachim Grunewald remarks that the addition of the Frobenius operators is not sufficient for finite generation in the case of $NA(point)$.

Reference: [GKM]

SP. (Stratos Prassidis, Canisius)

How does the Bass-Heller-Swan type splitting for Wh-theory, given by Hughes-Prassidis, relate to the H utteman-Klein-Vogell-Waldhausen-Williams splitting for A-theory?

References: [HP00] [HKV$^+$01]

JD1. (Jim Davis, Indiana)

What is a computation of $L_*(\mathbb{Z}[x, y])$?

JD2. (Jim Davis, Indiana)

Formulate the Farrell-Jones isomorphism conjecture in L-theory with respect to the family $CFIN = CYCLICS \cup FINITES$, using twisted transfers to eliminate the types of virtually cyclic groups which surject onto D_∞.

QK. (Qayum Khan, Vanderbilt)

What is a computation of $L_*(\mathbb{Z}[C_4 *_{C_2} C_4])$? Observe that the $\mathbb{Z}[C_2]$-bimodule structure on $\mathbb{Z}[C_4 - C_2]$ is standard but the involution is non-standard.

References

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