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ON FREELY DECOMPOSABLE MAPPINGS OF CONTINUA

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Abstract. We introduce and study a generalization of monotone mappings called freely decomposable mappings. Among the results established are the following: (1) The limit of an inverse sequence of locally connected (semi-locally) connected continua with freely decomposable bonding mappings is cloally connected (semi-locally connected). (2) Every freely decomposable mapping onto a locally connected continuum without separating points is monotone. (3) Every freely decomposable mapping on a locally connected unicoherent continuum is monotone. (4) Every freely decomposable mapping onto [0,1] is confluent.

1. Introduction

The purpose of this paper is to introduce and study a generalization of monotone mappings called *freely decomposable* mappings. This class of mappings shares with the monotone mappings the property of preserving local connectedness in inverse limits. More precisely, it will be shown that every inverse sequence of locally connected continua with freely decomposable bonding mappings has a locally connected limit.

The fact that monotone mappings preserve local connectedness in inverse limits was established in [1]. The more general problem of determining when the limit of an inverse sequence of locally connected continua is locally connected has been studied in [4] and [6]. However, the authors are unaware of any previously studied generalization of monotone mappings which preserves local connectedness in inverse limits.

Some of the most basic facts about freely decomposable mappings are stated in the abstract. Freely decomposable mappings on several special types of continua are considered in the final section.

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2. Preliminaries

A *continuum* is a compact connected metric space and a mapping is a continuous surjection.

Throughout the paper $f\colon X \to Y$ will denote a mapping of continua.

If X is a continuum, then $X = A \cup B$ is a *decomposition* provided that A and B are proper subcontinua of X. The continuum X is *freely decomposable* if for each pair of distinct points a and b in X, there exists a decomposition $X = A \cup B$ such that $a \in A \setminus B$ and $b \in B \setminus A$.

We shall make frequent use of the fact that a continuum is semi--locally connected if and only if it is freely decomposable [8].

The continuum X is *aposyndetic* at p with respect to the subset K if there exists a subcontinuum H of X such that

$$p \in \mathrm{Int}(H) \subseteq H \subseteq X \setminus K.$$

If X is aposyndetic at p with respect to each subcontinuum of X not containing p, then X is said to be *continuum-aposyndetic* at p. If X is continuum-aposyndetic at each point, then X is said to be *continuum--aposyndetic*.

We define the continuum X to be *freely decomposable with respect to points and subcontinua* if for each subcontinuum C of X and each point $a \in X \setminus C$ there exists a decomposition $X = A \cup B$ such that $a \in A \setminus B$ and $C \subseteq B \setminus A$.

The following lemma was mentioned in [9] without proof. It is a consequence of Theorem 4 of [3]; see also Theorem 2 of [7].

LEMMA 1. *A continuum is locally connected if and only if it is continuum-aposyndetic.*

THEOREM 1. *The continuum X is locally connected if and only if it is freely decomposable with respect to points and subcontinua.*

Proof. If X is freely decomposable with respect to points and subcontinua, then X is continuum-aposyndetic, and hence locally connected by Lemma 1.

Now suppose that X is locally connected, C is a subcontinuum of X, and $a \in X \setminus C$. Using the fact that the decomposition space X/C is freely decomposable, it follows easily that there is a decomposition $X = A \cup B$ such that $a \in A \setminus B$ and $C \subseteq B \setminus A$.

3. Freely decomposable mappings

The mapping $f\colon X \to Y$ is said to be *freely decomposable* if for each decomposition $Y = A \cup B$ there exists a decomposition $X = A' \cup B'$ such that $f(A') \subseteq A$ and $f(B') \subseteq B$. If, for each decomposition $Y = A \cup B$, the sets $f^{-1}(A)$ and $f^{-1}(B)$ are connected, then f is said to be *strongly freely decomposable.*

Notation. Hereafter we shall refer to freely decomposable mappings as FD mappings and to strongly freely decomposable mappings as SFD mappings.

Let $f\colon X \to Y$ and $g\colon Y \to Z$ be mappings of continua. The following fact are immediate consequences of the definitions.

(1) If f is an SFD mapping, then f is an FD mapping.

(2) If f is a monotone mapping, then f is an SFD mapping.

(3) If f and g are FD mappings (SFD mappings), then $g \circ f$ is an FD mapping (SFD mapping).

(4) If $g \circ f$ is an FD mapping (SFD mapping), then g is an FD mapping (SFD mapping).

We now describe a method for constructing SFD mappings which yields numerous examples of non-monotone SFD mappings. Other specific examples of FD and FSD mappings will be given in the final two sections.

Let Y be a continuum, K a compactum, and $\pi\colon K \times Y \to Y$ the projection mapping. Let F be a non-empty proper closed subset of Y, and let $\mathscr{D}$ be the upper semi-continuous decomposition of $K \times Y$ whose non-degenerate elements are of the form $K \times \{y\}$ for $y \in F$. Let $X = (K \times Y)/\mathscr{D}$ and let $D\colon K \times Y \to X$ be the decomposition mapping. Define $f\colon X \to Y$ to be the unique mapping such that $\pi = f \circ D$.

It is easy to verify that if C is subcontinuum of Y such that $C \cap F \neq \emptyset$, then $f^{-1}(C)$ is connected. Consequently f is an SFD mapping whenever the following condition is satisfied:

(*) For each decomposition $Y = A \cup B$, the sets $A \cup F$ and $B \cup F$ are non-empty.

A simple example which satisfies (*) occurs when Y is a finite tree and F contains the end points of Y. In particular, if $Y = [-1, 1]$ and $F = \{-1, 1\}$, then X is just the suspension of K and f is the projection of X onto $[-1, 1]$.

Notice that the mapping f is non-monotone whenever K is not connected.

4. Inverse limits with freely decomposable bonding mappings

We begin with a theorem which characterizes semi-local connectedness and local connectedness in terms of the existence of certain FD mappings.

Let $\mathscr{F}$ be a family of mappings on a continuum X with arbitrary continua as images. The family $\mathscr{F}$ is said to *separate points* if for each pair of distinct points a and b in X there exists an f in $\mathscr{F}$ such that $f(a) \neq f(b)$. The family $\mathscr{F}$ is said to *separate points from subcontinua* if for each subcontinuum C of X and each point $a \in X \setminus C$ there exists an f in $\mathscr{F}$ such that $f(a) \notin f(C)$.

THEOREM 2. *Let $\mathscr{F}$ bea famil y of* FD *mappings on a continuum X.*

(1) *If $\mathscr{F}$ separates points and* Im (f) *is semi-locally connected for each f in $\mathscr{F}$, then X is semi-locally connected.*

(2) *If $\mathscr{F}$ separates points from subcontinua and* Im (f) *is locally connected for each f in $\mathscr{F}$, then X is locally connected.*

Proof. (1) It suffices to show that X is freely decomposable. Let $a \neq b$ in X and choose f in $\mathscr{F}$ such that $f(a) \neq f(b)$. Let $\operatorname{Im}(f) = A \cup B$ be a decomposition such that $f(a) \in A \setminus B$ and $f(b) \in B \setminus A$. Let $X = A' \cup B'$ be a decomposition such that $f(A') \subseteq A$ and $f(B') \subseteq B$. Now $a \in A' \setminus B'$ and $b \in B' \setminus A'$ as required.

(2) According to Theorem 1 it sufficies to show that X is freely decomposable with respect to points and subcontinuia. The proof is analogous to that for (1).

In what follows let (X_n, f_n) be an inverse sequence of continua with limit X and projection mappings $\pi_n: X \to X_n$. The reader is referred to [1] for basic facts about inverse limits.

LEMMA 2. *The bonding mappings f_n are* FD *mappings if and only if the projection mappings π_n are* FD *mappings.*

Proof. Since π_n is an FD mapping, then since $\pi_n = f \circ \pi_{n+1}$ it follows that f_n is an FD mapping.

Now suppose that each bonding mapping is an FD mapping. Given m we must show that π_m is an FD mapping. Let $X_m = A_m \cup B_m$ be a decomposition and choose a decomposition $X_{m+1} = A_{m+1} \cup B_{m+1}$ such that $f_m(A_{m+1}) \subseteq A_m$ and $f_m(B_{m+1}) \subseteq B_m$. Continue this process inductively to obtain a sequence of decompositions $X_{m+k} = A_{m+k} \cup B_{m+k}$ such that $f_{m+k-1}(A_{m+k}) \subseteq A_{m+k-1}$ and $f_{m+k-1}(B_{m+k}) \subseteq B_{m+k-1}$. Let A denote the limit of $(A_{m+k}, f_{m+k}|A_{m+k+1})$ and let B denote the limit of $(B_{m+k}, f_{m+k}|B_{m+k+1})$. One can verify that $X = A \cup B$ is a decomposition, $\pi_m(A) \subseteq A_m$, and $\pi_m(B) \subseteq B_m$. Thus π_m is an FD mapping.

THEOREM 3. *Let (X_n, f_n) be an inverse sequence of continua with* FD *bonding mappings and limit X.*

(1) *If each X_n is semi-locally connected, then X is semi-locally connected.*

(2) *If each X_n is locally connected, then X is locally connected.*

Proof. It suffices to observe that $\{\pi_n\}$ s a family of FD mappings on X which separates points from subcontinua. The desired conclusions follow immediately from Theorem 2.

5. Some basic properties of freely decomposable mappings

The mapping $f\colon X \to Y$ is said to be *confluent* [2] if for each subcontinuum C of Y, every component of $f^{-1}(C)$ is mapped by f onto C.

THEOREM 4. *If $f\colon X \to Y$ is a confluent* FD *mapping, then f is an* SDF *mapping.*

Proof. Let $Y = A \cup B$ be a decomposition, and let $X = A' \cup B'$ be a decomposition such that $f(A') \subseteq A$ and $f(B') \subseteq B$. Denote by Q the component of $f^{-1}(A)$ which contains A'. It suffices to show that $Q = f^{-1}(A)$. If not, there exists a component Q' of $f^{-1}(A)$ distinct from Q. Thus $Q' \subseteq B'$, and consequently $f(Q') \subseteq B$ which contradicts the confluence of f.

THEOREM 5. *If $Y = [0, 1]$ and $f\colon X \to Y$ is an* FD *mapping, then f is confluent.*

Proof. Suppose that f is not confluent. There exist a subinterval $[a, b]$ of $[0, 1]$ and a component Q of $f^{-1}([a, b])$ such that $f(Q) \neq [a, b]$. Assume without loss of generality that $0 \neq a$ and $b \notin f(Q)$. Let $\{x_n\}$ be an increasing sequence in $Y \setminus \{0, a\}$ converging to a. For each n let $X = A_n \cup B_n$ be a decomposition such that $f(A_n) \subseteq [0, x_n]$ and $f(B_n) \subseteq [x_n, 1]$. Since $B_{n+1} \subseteq B_n$ and $Q \subseteq B_n$, it follows that $B = \cap \{B_n\}$ is a continuum containing Q and $f(B) = [a, 1]$. Now $f^{-1}([b, 1]) \subseteq B \setminus Q$. Consequently, there is a subcontinuum Q' of X such that $Q \subseteq Q' \subseteq B \setminus f^{-1}([b, 1])$ and $Q \neq Q'$. Thus $f(Q') \subseteq [a, b]$ contradicting the fact that Q is a component of $f^{-1}([a, b])$.

THEOREM 6. *If Y is locally connected and $f\colon X \to Y$ is an* SFD *mapping, then f is confluent.*

Proof. Suppose that f is not confluent. There exists a subcontinuum C of Y and a component Q of $f^{-1}(C)$ such that $f(Q) \neq C$. Let $a \in C \setminus f(Q)$. By Theorem 1 there is a decomposition $Y = A \cup B$ such that $a \in A \setminus B$ and $f(Q) \subseteq B \setminus A$. Now $f^{-1}(A \cup C)$ is a continuum containing Q and $f^{-1}(A)$ and $Q \cap f^{-1}(A) = \emptyset$. Let Q' be a subcontinuum of X such that $Q \subseteq Q' \subseteq f^{-1}(A \cup C) \setminus f^{-1}(A)$ and $Q \neq Q'$. Thus $f(Q') \subseteq C$ contradicting the fact that Q is a component of $f^{-1}(C)$.

Example 1. Let X be an equilateral triangle, let Y be a simple triod inscribed in X with its end points at the vertices of X, and let $f\colon X \to Y$ be a mapping which »collapses« X onto Y. Then f is an FD mapping which is not confluent (hence not SFD).

LEMMA 3. *Suppose that Y is locally connected and that $f\colon X \to Y$ is an* FD *mapping. If y is a non-separating point of Y, then $f^{-1}(y)$ is connected.*

Proof. According to ([11], p. 50) there exist arbitrarily small connected open sets about y whose complements are connected. Thus there is a seqeunce of decompositions $Y = A_n \cup B_n$ such that $y \in A_{n+1} \subseteq A_n \setminus B_n$ for each n, and $\{y\} = \cap \{A_n\}$. Let $X = A_n \cup B_n$ be a sequence of decompositions such that $f(A_n') \subseteq A_n$ and $f(B_n') \subseteq B_n$. Since $A_1' \supseteq A_2' \supseteq \ldots \supseteq A_n' \supseteq \ldots$, the set $f^{-1}(y) = \cup \{A_n'\}$ is a continuum.

THEOREM 7. *Let Y be a locally connected continuum which contains no separating points. If $f\colon X \to Y$ is an* FD *mapping then f is monotone.*

COROLLARY 1. *If Y is a manifold (with or without boundary) and $Y \not\approx [0, 1]$, then every* FD *mapping $f\colon X \to Y$ is monotone.*

Let X be a continuum. The following set valued functions were defined in [8].

$K(x) = \{y \in X\colon$ X is not aposydentic at x with respect to $y\}$

$L(x) = \{y \in X\colon$ X is not aposyndetic at y with respect to $x\}$.

THEOREM 8. *If Y is semi-locally connected and $f\colon X \to Y$ is an* FD *mapping, then $f(K(x)) = f(x)$ and $f(L(x)) = f(x)$ for each x in X.*

Proof. Suppose that $y \in K(x)$ and $f(x) \neq f(y)$. Since Y is freely decomposable there is a decomposition $Y = A \cup B$ such that $f(x) \in A \setminus B$ and $f(y) \in B \setminus A$. Let $X = A' \cup B'$ be a decomposition such that $f(A') \subseteq A$ and $f(B') \subseteq B$. Thus

$$x \in A' \setminus B' \subseteq \operatorname{Int}(A') \subseteq A' \subseteq X \setminus \{y\},$$

so X is aposyndetic at x with respect to y which is a contradiction. Consequently, $f(x) = f(y)$ as desired.

The proof that $f(L(x)) = f(x)$ is similar.

COROLLARY 2. *Suppose that X is a continuum whit the property that for each pair of points x and y either $K(x) \cap K(y) \neq \emptyset$ or $L(x) \cap L(y) \neq \emptyset$. Then every* FD *mapping from X onto a semi-locally connected continuum is constant.*

6. Freely decomposable mappings on some special types of continua

The continuum X is *unicoherent* if for each decomposition $X = A \cup B$ the set $A \cup B$ is connected. If each subcontinuum of X is unicoherent, X is said to be *hereditarily unicoherent*. A *dendroid* is an arcwise connected hereditarily unicoherent continuum. A *dendrite* is a locally connected hereditarily unicoherent continuum.

The next theorem is an immediate consequence of the definitions.

THEOREM 9. *If X is unicoherent and $f: X \to Y$ is an* SFD *mapping, then Y is unicoherent.*

Example 2. FD mappings need not preserve unicoherence. To see this, let X denote the cone over the compactum $\{x \in [-1, 1]: x = 1, x = -1, \text{ or } x = \pm n/(n+1) \text{ for some natural number } n\}$. Let $Y = X/\{-1, 1\}$, and let $f: X \to Y$ be the decomposition mapping.

THEOREM 10. *If X is unicoherent, Y is locally connected, and $f: X \to Y$ is an* FD *mapping, then f is monotone.*

Proof. By Lemma 3 it suffices to show that $f^{-1}(y)$ is connected in case $Y \setminus \{y\}$ is not connected. Since Y is locally connected, there are at most countably many components, say Q_n, of $Y \setminus \{y\}$. For each n let g_n be the monotone mapping of Y onto the locally connected continuum $Q_n \cup \{y\}$ defined by

$$g_n(x) = \begin{cases} x \text{ if } x \in Q_n \text{ and} \\ y \text{ if } x \in Y \setminus Q_n. \end{cases}$$

Since y is a non-separating point of $Q_n \cup \{y\}$ and $g_n \circ f$ is an FD mapping, it follows from Lemma 3 that $f^{-1}(g_n^{-1}(y))$ is connected for each n. It is easily seen that $f^{-1}(y) = \cap \{f^{-1}(g_n^{-1}(y))\}$. Let $H_1 = f^{-1}(g_1^{-1}(y))$ and inductively define $H_n = f^{-1}(g_n^{-1}(y)) \cap H_{n-1}$. Then $f^{-1}(y) = \cap \{H_n\}$ and $H_{n+1} \subseteq H_n$ for each n. But by the unicoherence of X, each H_n is connected. Thus $f^{-1}(y)$ is connected as required.

COROLLARY 3. *Every* FD *mapping on a dendrite is monotone.*

FD mappings on dendroids need not be monotone (see Example 2).

THEOREM 11. *If X is hereditarily unicoherent, Y is semi-locally connected, and $f: X \to Y$ is an* FD *mapping, then f is monotone. Consequently, Y is adendrite.*

Proof. Suppose that f is not monotone and let y be a point of Y for which $f^{-1}(y)$ is not connected. Let Q_1 and Q_2 be distinc components of $f^{-1}(y)$ and let I be a subcontinuum of X which is irreducible from Q_1 to Q_2 (i. e., I meets Q_1 and Q_2 but no proper subcontinuum of I does). Let $x \in I \setminus f^{-1}(y)$. There is a decomposition $Y = A \cup B$ such that $y \in A \setminus B$ and $f(x) \in B/A$. Let $X = A' \cup B'$ be a decomposition such that $f(A') \subseteq A$ and $f(B') \subseteq B$. Now $Q_1 \cup Q_2 \subseteq A'$ and $x \notin A'$. Thus, by hereditary unicoherence, $A' \cup I$ is a proper subcontinuum of I which meets Q_1 and Q_2. This contradicts the fact that I is irreducible Q_1 to Q_2.

Since monotone mappings preserve hereditary unicoherence, Y is a dendrite (e. g., Corollary 2.1 of [5]).

Example 3. Let X denote the »sin $1/x$ curve« and let a and b be the end points of the limit arc. Let $Y = X/\{a, b\}$ and let $f: X \to Y$ denote the decomposition mapping. Then X is hereditarily unicoherent and f is an SFD mapping, yet f is not monotone. Thus Theorem 11 fails without the assumption that Y be semi-locally connected.

The continuum X is said to be *irreducible* if there exist points a and b of X such that no proper subcontinuum of X contains a and b. The continuum X is *indecomposable* if there does not exist a decomposition $X = A \cup B$.

It is well-known that every indecomposable continuum is irreducible (e. g., [10], p. 212).

The next two theorems are immediate consequences of the definitions.

THEOREM 12. *If X is indecomposable and $f: X \to Y$ is an* FD *mapping, then Y is indecomposable.*

THEOREM 13. *If Y is indecomposable and $f: X \to Y$ is any mapping, then f is an* SFD *mapping.*

Example 4. Although indecomposability is preserved by FD mappings, irreducibility is not. To see this, let I^+ denote the familiar indecomposable »buckethandle« continuum lying in the plane as described in ([10], p. 204). Denote by I^- the reflection of I^+ across the y-axis and let $X = I^+ \cup I^-$. Define $f: X \to Y$ to be the decomposition mapping which identifies points of the form (x, y) and $(-x, y)$ whenever $0 < x < \frac{1}{2}$. One can verify that X is irreducible and f is an FD mapping, but Y is not irreducible.

We now establish that irreducibility is preserved by SFD mappings and by FD mappings onto locally connected continua.

THEOREM 14. *If X is irreducible and $f: X \to Y$ is an* SFD *mapping, then Y is irreducible.*

Proof. Suppose that X is irreducible from a to b and that Y is not irreducible from $f(a)$ to any other point. It follows (see [10], Theorem 4, p. 192) that there is a decomposition $Y = A \cup B$ such that $f(a) \in A \cap B$. Now $X = f^{-1}(A) \cup f^{-1}(B)$ is a decomposition and without loss of generality, $b \in f^{-1}(B)$. Thus $f^{-1}(B)$ contains a and b contradicting the rireducibility of X.

THEOREM 15. *If X is irreducible, Y is a nondgenerate locally connected continuum, and $f: X \to Y$ is an* FD *mapping, then f is monotone. Consequently, Y is an arc.*

Proof. Suppose that X is irreducible from a to b. We begin by showing that Y is an arc with end points $f(a)$ and $f(b)$. If not, let C be a proper subcontinuum of Y containing $f(a)$ and $f(b)$. Let $p \in Y \setminus C$. By Theorem 1 there is a decomposition $Y = A \cup B$ with $p \in A \setminus B$ and $C \subseteq B \setminus A$. Chose a decomposition $X = A' \cup B'$ such that $f(A') \subseteq A$ and $f(B') \subset B$. Thus, a and b belong to B' which is a contradiction.

Assume that the arc Y is ordered form $f(a)$ to $f(b)$. Notice that f is an SFD mapping by Theorems 4 and 5. Furthermore, $f^{-1}(f(a))$ and $f^{-1}(f(b))$ are connected by Lemma 3. If f is not monotone, there exists a t in Y such that $f(a) < t < f(b)$ and $f^{-1}(t)$ is not connected. Let Q be a component of $f^{-1}(t)$, let Q_1 be a subcontinuum of $f^{-1}([f(a), t])$ which properly contains Q but does not contain $f^{-1}(t)$, and let Q_2 be a subcontinuum of $f^{-1}([t, f(b)])$ which properly contains Q but does not contain $f^{-1}(t)$. Then $f(Q_1 \cup Q_2) = [t_1, t_2]$ where $t_1 < t < t_2$. Now $f^{-1}([f(a), t_1]) \cup Q_1 \cup Q \cup f^{-1}([t_2, f(b)])$ is a subcontinuum of X which contains a and b, but does not contain $f^{-1}(t)$. This contradicts the fact that X is irreducible from a to b. Consequently f is monotone.

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O SLOBODNO RASTAVLJIVIM PRESLIKAVANJIMA KONTINUUMA

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Sadržaj

U članku se uvodi i proučava jedna generalizacija monotonih preslikavanja koja su nazvana slobodno rastavljiva preslikavanja. Medu dokaznim rezultatima su ovi: (1) Limes inverznog niza lokalno povezanih (semi-lokalno povezanih) kontinuuma sa slobodno rastavljivim veznim preslikavanjima je lokalno poveazn (semi-lokalno povezan). (2) Svako slobodno rastavljivo preslikavanje na lokalno povezan kontinuum bez separirajućih točaka je monotono. (3) Svako slobodno rastavljivo preslikavanje na lokalno povezan unikoherentan kontinuum je monotono. (4) Svako slobodno rastavljivo preslikavanje na [0, 1] je konfluentno.