Math 372B: Seminar in Topology
Manifold Topology

Bruce Hughes

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1 Introduction

Meeting time and location: TBA

Brief Description: We will begin by studying cobordism as a generalized homology theory. Then we will cover ordinary homology and cohomology from the bordism point of view. We will follow Kreck's approach in which manifold cycles are replaced by stratifolds. This will allow us to introduce Euler, Stiefel-Whitney, Chern, and Pontrjagin characteristic cohomology classes of vector bundles over manifolds. As an application we will sketch Milnor's construction of exotic smooth structures on seven dimensional spheres.

Surgery theory will be in the background of our exposition, and with the remaining time, we will go further into that subject. Surgery theory is one of the main tools in the classification of high dimensional manifolds. Our goal is to get to the point where we can understand aspects of one of the main problems in topology and geometry: the Novikov Conjecture. There are two different formulations of the Novikov Conjecture. One concerns the homotopy invariance of certain cohomology classes determined by the Pontryagin classes. The second concerns the injectivity of a certain assembly map in the surgery exact sequence.

One reason to be interested in trying to understand the transition from the characteristic class to the assembly map points of view is that two other sets of conjectures also concern assembly maps, namely, the Baum-Connes Conjectures and the Farrell-Jones Conjectures.

1.1 References

Here is a list of books you might find useful.

Texts

1. Matthias Kreck, *Differential Algebraic Topology*, 2002. To be distributed in class.

2. Andrew Ranicki, *Algebraic and Geometric Surgery*, Oxford University Press, 2003. Available in the bookstore.

History

1. John Dieudonné, *A History of Algebraic and Differential Topology, 1900–1960*, Birkhäuser, 1989.
2. I. M. James (ed.), *History of Topology*, North-Holand, 1999.

Basics on smooth differential manifolds

1. Dennis Barden and Charles Thomas, *An Introduction to Differential Manifolds*, Imperial College Press, 2003.
2. Glen E. Bredon, *Topology and Geometry*, Springer, 1993. (Chapter II)
3. T. Bröker and K. Jänich, *Introduction to Differential Topology*, Cambridge University Press, 1982.
4. L. Conlon, *Differentiable Manifolds*, Second edition, Birkhäuser, 2001.
5. Victor Guillemin and Alan Pollack, *Differential Topology*, Prentice-Hall, 1974.
6. Morris W. Hirsch, *Differential Topology*, Springer, 1976.
7. Serge Lang, *Differential and Riemannian Manifolds*, Springer, 1995.
8. John M. Lee, *Introduction to Smooth Manifolds*, Springer, 2003.
9. I. Madsen and J. Tornehave, *From Calculus to Cohomology*, Cambridge University Press, 1997.
10. John W. Milnor, *Topology from the Differentiable Viewpoint*, Princeton University Press, 1997.

More advanced books on manifolds and other topics related to this course

1. Pierre Euclide Conner and Edwin Earl Floyd, *Differential Periodic Maps*, Springer, 1964. In spite of the title, the first half of this book is about cobordism.
2. F. T. Farrell, *Surgical Methods in Rigidity*, Springer, 1996.
3. Allen Hatcher, *Vector Bundles and K-Theory*, avaiable at

   `http://www.math.cornell.edu/~hatcher/VBKT/VBpage.html`

4. Antoni A. Kosinski, *Differential Manifolds*, Academic Press, 1993.

5. Wolfgang Lück, *A Basic Introduction to Surgery Theory*, available at

   `http://www.ictp.trieste.it/~pub_off/lectures/vol9.html`

6. Wolfgang Lück and Matthias Kreck, *The Novikov Conjecture*, Birkhäuser, 2005. See the introduction at

   `http://www.math.uni-muenster.de/u/lueck/org/staff/publications.html#bo`

7. John W. Milnor and James D. Stasheff, *Characteristic Classes*, Princeton University Press, 1974.

Various approaches to algebraic topology

1. Marcelo Aguilar, Samuel Gitler and Carlos Prieto, *Algebraic Topology from a Homotopical Viewpoint*, Springer, 2002.

2. Raoul Bott and Loring W. Tu, *Differential Forms in Algebraic Topology*, Springer, 1982.

3. J. Davis and P. Kirk, *Leture Notes in Algebraic Topology*, American Mathematical Society, 2001.

4. Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002. Available at

   `http://www.math.cornell.edu/~hatcher/AT/ATpage.html`

5. J. Peter May, *A Concise Course in Algebraic Topology*, University of Chicago Press, 1999. Available at

   `http://www.math.uchicago.edu/%7Emay/`

6. Hajime Sato, *Algebraic Topology: An Intuitive Approach*, American Mathematical Society, 1999.

7. R. M. Switzer, *Algebraic Topology. Homotopy and Homology*, Springer, 1975.

Collections of surveys and expositions

1. S. Cappell, A. Ranicki, and J. Rosenberg (eds.), *Surveys on Surgery Theory. Volumes 1 and 2*, Princeton University Press, 2001.

2. R. Daverman and R. Sher (eds.), *Handbook on Geometric Topology*, Elsevier, 2001.

3. F. T. Farrell, L. Goettshe, and W. Lueck (eds.), *Topology of High Dimensional Manifolds*, ICTP Lecture Notes, 2002. Available at

http://www.ictp.trieste.it/~pub_off/lectures/vol9.html

4. S. Ferry, A. Ranicki, and J. Rosenberg (eds.), *Novikov Conjectures, Index Theorems and Rigidity. Volumes 1 and 2*, Cambridge University Press, 1995.

5. F. Quinn (ed.), *Prospects in Topology*, Princeton University Press, 1995.

6. A. A. Ranicki (ed.), A. J. Casson, D. P. Sullivan, M. A. Armstrong, C. P. Rourke, and G. E. Cooke, *The Hauptvermutung Book. A collection of papers on the topology of manifolds*, Kluwer, 1996.

Advanced books related to this course

1. S. Buoncristiano, *Fragments of Geometric Topology from the Sixties*, Geometry & Topology Publications, 2003. Available at

http://www.maths.warwick.ac.uk/gt/gtmcontents6.html

2. B. Hughes and A. Ranicki, *Ends of Complexes*, Cambridge University Press, 1996.

3. F. T. Farrell and L. E. Jones, *Classical Aspherical Manifolds*, American Mathematical Society, 1990.

4. A. A. Ranicki, *Algebraic L-Theory and Topological Manifolds*, Cambridge University Press, 1992.

5. C. T. C. Wall, *Surgery on Compact Manifolds*, second edition edited by A. A. Ranicki, American Mathematical Society, 1999.

6. S. Weinberger, *The Topological Classification of Stratified Spaces*, University of Chicago Press, 1994.

7. S. Weinberger, *Computers, Rigidity, and Moduli: The Large Scale Fractal Geometry of Riemannian Moduli Space*, Princeton University Press, 2004.