

Review Sheet

Exam I

Chapter 1

- Definitions and Terminology
 - Ordinary differential equation vs. partial differential equation
 - Order of a differential equation
 - Linear vs. Nonlinear
 - Solution
 - What does it mean for a function to be a solution of a differential equation?
 - What is the interval of validity of a solution?
 - What is the difference between an explicit and an implicit solution?
- Initial-Value Problems
 - Existence and uniqueness of a solution
 - Theorem 1.1: Existence and uniqueness for nonlinear IVPs
 - Continuity of f and $\partial f/\partial y$
 - Inclusion of initial point (x_0, y_0)
- Applications of Differential Equations
 - Population dynamics
 - Radioactive decay
 - Mixtures, i.e., tank problems
 - Series circuits

Chapter 2

- Phase Portraits
 - Critical points, i.e., equilibrium points
 - Concavity changes
 - Graphing the solution without solving the differential equation
- Separation of Variables
- Linear equations
 - Homogeneous vs. nonhomogeneous
 - Integrating factor
 - What is a general solution?
- Exact equations
 - Definition of exact equation
 - How do we solve an exact equation?
(Note: Solutions are usually written implicitly as an equation set equal to a constant)
- Homogeneous equations
 - Solution by substitution $u=y/x$, which reduces the equation to a separable equation

Chapter 3

- Further Applications of Differential Equations
 - Population dynamics
 - Bacterial growth
 - More mixtures, i.e., tank problems
 - More series circuits

Chapter 4

- Higher order linear equations
 - Theorem 4.1: Existence and uniqueness of solutions
 - Continuity of coefficients
 - Leading coefficient nonzero
 - Inclusion of initial point x_0 in largest open interval where all coefficients are continuous and leading coefficient is nonzero
 - Principle of Superposition
 - Linear independence
 - Fundamental set of solutions
 - Homogeneous vs. nonhomogeneous
 - General solution
- Reduction of Order
 - Given one solution of a homogeneous differential equation, find a second by assuming $y_2 = u(x)y_1$
- Homogeneous Linear Equations with Constant Coefficients
 - Characteristic equation
 - Case 1: Distinct real roots
 - Case 2: Repeated real roots
 - Case 3: Complex conjugate roots
 - Higher-Order Constant Coefficient Equations
- Method of Undetermined Coefficients
 - Assumptions:
 - Constant coefficient equation
 - Nonhomogeneous term composed of only exponentials, sines, cosines and polynomials
 - How do we make a good guess?
 - What if our guess is part of the solution to the corresponding homogeneous problem?
- Variation of Parameters
 - If we know two solutions of a second order homogeneous equation, find a solution of the *nonhomogeneous* equation by assuming
$$Y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$$
- Cauchy-Euler Equations
 - Auxiliary equation
 - Case 1: Distinct real roots
 - Case 2: Repeated real roots
 - Case 3: Complex conjugate roots