

Vanderbilt University, Department of Mathematics, Spring 09

## **Math 394 – Compressed Sensing**

Time: TR 11:00-12:15, Room: SC 1310

**Instructor:** Simon Foucart, SC 1533, [simon.foucart@vanderbilt.edu](mailto:simon.foucart@vanderbilt.edu)

**Office hours:** TR 10:00–11:00, or by appointment

**Course web page:** [www.math.vanderbilt.edu/~foucart/teaching](http://www.math.vanderbilt.edu/~foucart/teaching)

**Text:** Comprehensive lecture notes will be distributed in class.

**Course outline:** Compressed Sensing — aka Compressive Sampling — may be described as a collection of concrete protocols for sensing and compressing data simultaneously. This exciting new field originates from the realization that signals and images can be faithfully reconstructed from what appears to be highly incomplete sets of measurements. The course will provide a detailed mathematical coverage of the field. Topics to be thoroughly studied include theoretical limitations,  $\ell_1$ -recovery algorithms, restricted isometry properties, Gaussian random matrices, adjacency matrices of expander graphs, widths, greedy recovery algorithms.

**Course structure:** The course will be 2/3 lecture-style and 1/3 seminar-style. As required work, it is requested to attend classes and complete homework, and to read a set of research papers and summarize them in a coherent 5/10-page essay to be presented in class. You may chose your own subject, or select one of the following:

- 1/ Applications of Compressed Sensing
- 2/ Reconstruction Algorithms
- 3/ Extensions of Compressed Sensing
- 4/ Connections with Coding and Information Theory
- 5/ Connections with High-Dimensional Geometry
- 6/ Connections with Machine Learning
- 7/ Connections with Finite Rate of Innovation
- 8/ Random Matrices
- 9/ Open Problems

## **Tentative Schedule:**

Lecture 1: Motivations and Applications

Lecture 2: Theoretical Limitations

Lecture 3: Reed–Solomon Decoding

Lecture 4:  $\ell_q$ -Strategy — Null-Space Property

Lecture 5:  $\ell_q$ -Strategy — Stability, Robustness

Lecture 6: A Primer on Convex Optimization

Lecture 7: Coherence and Exact Recovery by  $\ell_1$ -Minimization

Lecture 8: Restricted Isometry Property and Exact Recovery by  $\ell_1$ -Minimization

Lecture 9: Construction of Deterministic Sensing Matrices

Lecture 10: Restricted Isometry Property for Random Matrices

Lecture 11: Restricted Isometry Property for Gaussian Random Matrices

Lecture 12: Stable and Robust Recovery by  $\ell_q$ -Minimization

Lecture 13: Widths

Lecture 14: Shortcomings of the Restricted Isometry Property

Lecture 15: Using Expander Graphs

Lecture 16: Reweighted  $\ell_1$ -Algorithms

Lecture 17: Thresholding and Orthogonal Matching Pursuit

Lecture 18: ROMP and CoSamp

Lecture 19: Additional Topic

Lecture 20: Additional Topic