Math 198, Section 2, Test 1

February 6, 2008

Name:

Pledged

Honor code: I have neither given nor received help on this test.

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1. (15 pts) Give the explicit form of the solution to the initial value problem

$$(e^x + e^{-x})\frac{dy}{dx} = 1 + y^2 \qquad \text{subject to } y(0) = 0.$$

Determine the largest interval on which the solution is defined.
Hint: You may find useful to know that

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}.$$

2. (30 pts) Solve the differential equations

(a) $3\dfrac{dy}{dx} + 12y = 4$

(b) $\dfrac{dP}{dt} + 2tP = P + 4t - 2$

(c) $y\,dx = (ye^y - 2x)\,dy$

3. (20 pts) Explain why a solution to the initial value problem

$$(1 - y + 3x)\,\frac{dy}{dx} + 3y = x - 1, \qquad y(2) = 1,$$

exists and is unique on some interval $(2 - h, 2 + h)$, $h > 0$. You should state clearly any theorem that you use. Find this solution in implicit form.

4. (15 pts) A thermometer reading $50°C$ is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer reads $100°C$ after 1 minute and $150°C$ after 2 minutes. How hot is the oven? (Note: the original version above contained a misprint: the initial temperature should be $40°C$ instead of $50°C$.)

5. (20 pts) Using the method of your choice, solve the differential equation

$$\frac{dy}{dx} = \frac{-x+y}{3x+y}.$$

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