

Math 198: Practice Sheet for Test 2

1 Broad guidelines

You are expected to know the content of the course and to be able to answer any question from the assigned homework. In addition, you should be in a position to solve any exercise from the following non-exhaustive list:

8 p.138; 16,23,41 p.147-148; 19,37,33 p.158; 18 p.172; 8,14,37 p.178; 11,32 p.207-209; 16 p.231.

2 Practice test

You can assess your progress with the following exam-like questions. If you are well prepared, it should take you a little over 2 hours to complete.

Note: these are just training exercises, the content of Test 2 will of course be different.

1. Solve the differential equation

$$y'' - 2y' + 2y = \exp(2x)(\cos(x) - 3\sin(x)).$$

2. A mass weighing 12 pounds stretches a spring 2 feet. The mass is initially released from a point 1 foot below the equilibrium position with an upward velocity of 4 ft/s.
 - (a) Find the equation of the motion.
 - (b) What are the amplitude, period, and frequency of the simple harmonic motion?
 - (c) At what time does the mass return to its starting point?
 - (d) At what times is the velocity zero?
 - (e) What is the velocity of the mass at $t = 3\pi/16$?

3. Solve the initial-value problem

$$y'' - 3y = 0 \quad \text{subject to} \quad y(0) = 1 \text{ and } y'(0) = 5.$$

4. Consider the Cauchy–Euler equation

$$x^2 y'' - xy' + 2y = 0.$$

(a) Give a fundamental set of solutions on the interval $(0, \infty)$.

(b) Knowing that $y_1 = x \sin(\ln x)$ is one function in this set, obtain the other one using the method of reduction of order.

5. Solve the differential equation

$$y'' - 2y' + y = \exp(x) \arctan(x).$$

6. Write down the general solution of the differential equation

$$y^{(4)} - y'''' - 2y'' = 0.$$

7. A mass $m = 1$ kg is attached to a spring whose constant is $k = 16$ N·m⁻¹. The entire system is submerged in a liquid that imparts a damping force with damping constant $\beta = 10$ N·s·m⁻¹. Determine the equation of the motion if

(a) the mass is initially released from rest from a point 1 m below the equilibrium position,

(b) the mass is initially released with an upward velocity of 12 m·s⁻¹ from a point 1 m below the equilibrium position.

8. (a) Write the general solution of the differential equation

$$y^{(4)} - 2y'' + y = 0.$$

only in terms of hyperbolic functions.

(b) Give a particular solution of the differential equation

$$y^{(4)} - 2y'' + y = \sinh(x).$$

9. Solve the differential equation

$$x^2 y'' - 2xy' + 2y = x^4 \exp(x).$$

10. Use the method of undetermined coefficients to solve the differential equation

$$y'' - 2y' + 5y = \exp(x) \cos(2x).$$

11. Given that $y_1 = \cos(\ln x)$, $y_2 = \sin(\ln x)$ are two linearly independent solutions on $(0, \infty)$ of the differential equation

$$x^2 y'' + xy' + y = 0,$$

find the solution on $(0, \infty)$ of the initial-value problem

$$x^2 y'' + xy' + y = \sec(\ln x) \quad \text{subject to} \quad y(1) = 1 \text{ and } y'(1) = 1.$$