Math 198, Section 4, Extra Credit Quiz

February 13, 2008

Name:

Pledged

Honor code: I have neither given nor received help on this quiz.

---

1. **(10 pts) Consider the differential equations**

$$xy' - 8y = 0 \qquad \text{and} \qquad xy' - 8y = 8.$$

Give their general solutions on the interval $(0, \infty)$. No justifications are necessary.

Of course, $y' - \frac{8}{x} y = 0$ is the homogeneous equation associated to $y' - \frac{8}{x} y = \frac{8}{x}$. Many ways to solve it: separation of variables, integrating factor, or best write directly the solution as

$y(x) = C\exp\left(-\int -\frac{8}{x}\,dx\right) = C\exp\left(8\ln|x|\right) = C\exp\left(\ln(x^8)\right)$

$\underline{y(x) = C x^8}$

Clearly, a particular solution of the non-homogeneous equation is $y(x) = -1$, so its general solution is $\underline{y(x) = C x^8 - 1}$

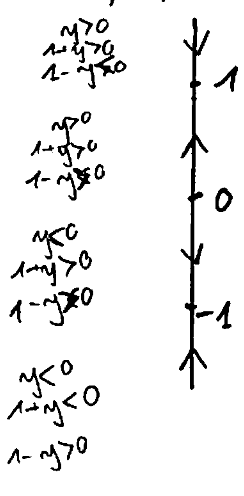
2. **(5pts) Find the critical points, draw the phase portrait, and determine which critical points are attractors/repellers of the differential equation**

$$\frac{dy}{dx} = y - y^3.$$

The critical points are the zeros of $y - y^3 = y(1+y)(1-y)$

Critical points: -1, 0, and 1

Phase portrait :



0 is a repeller

-1 and 1 are attractors

3. (10pts) Give the explicit form of the solution to the initial-value problem

$$\frac{dy}{dx} = (x+y+1)^2 \qquad \text{subject to} \qquad y(0) = -1.$$

Substitution $u = x+y+1$:

$\frac{du}{dx} = 1 + \frac{dy}{dx} = 1 + u^2$, $dx = \frac{du}{1+u^2}$

$x + C = \text{Arctan}(u) = \text{Arctan}(x+y+1)$

Initial condition : $0 + c = \text{Arctan}(0+(-1)+1) \Rightarrow \underline{c=0}$

Then: $x = \text{Arctan}(x+y+1)$

$x+y+1 = \tan(x)$

$\underline{y = \tan(x) - x - 1}$

4. (5 pts) State the precise statement of Theorem 1.1 on the existence of a unique solution to the initial-value problem

$$\frac{dy}{dx} = f(x,y) \qquad \text{subject to} \qquad y(x_0) = y_0.$$

<u>If</u> f and $\frac{\partial f}{\partial y}$ are continuous on some rectangular region $(a,b)\times(c,d)$ containing the point (x_0, y_0),

<u>then</u> there exists, for some $h>0$, an interval (x_0-h, x_0+h) contained in (a,b) on which a solution to the initial-value problem exists and is unique.