Math 198, Section 2, Extra Credit Quiz

February 13, 2008

Name:

Pledged

Honor code: I have neither given nor received help on this quiz.

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1. **(10 pts) Consider the differential equations**

$$xy' - 4y = 0 \qquad \text{and} \qquad xy' - 4y = -8.$$

Give their general solutions on the interval $(0, \infty)$. No justifications are necessary.

Of course, $y' - \frac{4}{x} y = 0$ is the homogeneous equation associated to $y' - \frac{4}{x} y = -\frac{8}{x}$. Many ways to solve it: separation of variables, integrating factor, or best write directly the solution as

$y(x) = c \exp\left(-\int -\frac{4}{x}\, dx\right) = c \exp\left(4 \ln|x|\right) = c \exp\left(\ln(x^4)\right)$

$\underline{y(x) = c\, x^4}$

Clearly, a particular solution of the non-homogeneous equation is $y(x) = 2$, so its general solution is $\underline{y(x) = c\,x^4 + 2}$

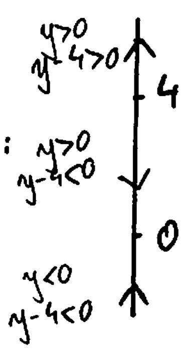
2. **(5pts) Find the critical points, draw the phase portrait, and determine which critical points are attractors/repellers of the differential equation**

$$\frac{dy}{dx} = y^2 - 4y.$$

The critical points are the zeros of $y^2 - 4y = y(y-4)$

<u>Critical points: 0 and 4</u>

Phase portrait:



<u>4 is a repeller</u>

<u>0 is an attractor</u>

3. **(10pts) Give the explicit form of the solution to the initial-value problem**

$$\frac{dy}{dx} = 2(2x+y+1)^2 \qquad \text{subject to} \qquad y(0) = -1.$$

Substitution $u = 2x+y+1$:

$$\frac{du}{dx} = 2 + \frac{dy}{dx} = 2 + 2u^2 \quad , \quad 2\,dx = \frac{du}{1+u^2}$$

$$2x + c = \operatorname{Arctan}(u) = \operatorname{Arctan}(2x+y+1)$$

Initial condition: $2\times0 + c = \operatorname{Arctan}(2\times0 + (-1) + 1) \Rightarrow \underline{c=0}$

Then: $2x = \operatorname{Arctan}(2x+y+1)$

$$2x+y+1 = \tan(2x)$$

$$\underline{y = \tan(2x) - 2x - 1}$$

4. **(5 pts) State the precise statement of Theorem 1.1 on the existence of a unique solution to the initial-value problem**

$$\frac{dy}{dx} = f(x,y) \qquad \text{subject to} \qquad y(x_0) = y_0.$$

<u>If</u> f and $\frac{\partial f}{\partial y}$ are continuous on some rectangular region $(a,b)\times(c,d)$ containing the point (x_0, y_0), <u>then</u> there exists, for some $h>0$, an interval (x_0-h, x_0+h) contained in (a,b) on which a solution to the initial-value problem exists and is unique.