Math 226 / CS 255, Spring 2007
Test 1

Name:

1. (20 pts) Prove that if a nonsingular matrix A admits a factorization $A = LU$ in which U and L are upper triangular and unit lower triangular square matrices, respectively, then L and U are unique.

2. (20 pts) For any $x_0 > -1$, the sequence (x_n) defined recursively by

$$x_{n+1} = 2^{n+1}\left[\sqrt{1 + 2^{-n}x_n} - 1\right]$$

converges to $\ln(1 + x_0)$. Arrange this formula in a way that avoids loss of significance.

3. (a) (10 pts) If A admits a Doolittle's factorization, what is a simple formula for the determinant of A?

   (b) (10 pts) Evaluate $P(x) = x^6 - 4x^4 + 2x^2 + 1$ at $x = 1/2$ by considering $P(x)$ as a polynomial in x^2 and using nested multiplication.

   (c) (15 pts) Convert the decimal number 55.4 to a binary number. Then convert the result into the hexadecimal system.

   (d) (10 pts) Assume that a computer requires 20 seconds to solve a 400×400 linear system using Gaussian elimination. Estimate the time needed to solve a 2000×2000 system.

4. (35 pts) Determine an LU-factorization of the matrix $A = \begin{bmatrix} 6 & 10 & 0 \\ 12 & 26 & 4 \\ 0 & 9 & 12 \end{bmatrix}$ in which L is a lower triangular matrix with 2's on its main diagonal.

5. (20 pts) Write a pseudocode for computing the machine epsilon.

6. (30 pts) Consider the matrix

$$A = \begin{bmatrix} 3 & -1 & 0 & \cdots & 0 \\ -1 & 3 & -1 & & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & -1 & 3 & -1 \\ 0 & \cdots & 0 & -1 & 3 \end{bmatrix}.$$

Does the Gauss–Seidel iteration converge? What is the explicit form of the associated iteration matrix $I - Q^{-1}A$ in the case $n = 2$?

7. Suppose that the square matrix $A = [a_{i,j}]_{i,j=1}^n$ is strictly diagonally dominant, that is to say $|a_{i,i}| > \sum_{j \neq i} |a_{i,j}|$ for all i. Prove that A is nonsingular [Hint: if $Ax = 0$ for some $x \neq 0$, you should derive a contradiction by looking at the k-th component of Ax, where $|x_k| = \max_i |x_i|$]. Prove furthermore that A admits a Doolittle's factorization. You should state clearly any theorem you use.

Pledged

Honor code: I have neither given nor received help on this test.