Math 226 / CS 255, Spring 2007
Take Home Test

Name:

1. Find both the Lagrange and Newton forms of the interpolating polynomial for the data $(-2,0)$, $(0,1)$, $(1,-1)$. Write the resulting polynomials in the form $a+bx+cx^2$ in order to verify that they are identical as functions.

2. Find the least-squares solution to the system

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}.$$

3. Determine whether the algorithm

   $x \leftarrow a_n b_n$
   for $i = 1$ to n
       do $x \leftarrow (x + a_{n-i})b_i$ **end do**
   end for

   computes $x = \sum_{i=0}^{n} a_i \prod_{j=0}^{i} b_j$.

4. Find the divided difference $[0,1,2,3,4]f$ for the function f which takes the values

$$f(0)=6, \quad f(1)=10, \quad f(2)=20, \quad f(3)=48, \quad f(4)=106.$$

   What is the degree of the interpolating polynomial of f at the points 0, 1, 2, 3, and 4?

5. Prove that the algorithm for computing the coefficients of the polynomial interpolant in Newton's form involves n^2 multiplications/divisions.

6. Verify that all the polynomials

$$\begin{aligned} p(x) &= 5x^3 - 27x^2 + 45x - 21 \\ q(x) &= x^4 - 5x^3 + 8x^2 - 5x + 3 \\ r(x) &= 2 + (-1 + (3 + 5(-3 + x))(-2 + x))(-1 + x) \end{aligned}$$

interpolate the values 2, 1, 6, and 47 at the points 1, 2, 3, and 4. Explain why it does not violate the uniqueness part of the existence theorem for polynomial interpolation.

7. The polynomial $p(x) = x^4 - x^3 + x^2 - x + 1$ takes the values

$$p(-2) = 31, \quad p(-1) = 5, \quad p(0) = 1, \quad p(1) = 1, \quad p(2) = 11, \quad p(3) = 61.$$

Find a polynomial q taking the values

$$q(-2) = 31, \quad q(-1) = 5, \quad q(0) = 1, \quad q(1) = 1, \quad q(2) = 11, \quad q(3) = 30.$$

This can be done with little work.

8. Use a computer to find a polynomial p of degree at most 10 that interpolates $|x|$ on $[-1, 1]$ at 11 equally spaced nodes. Print the difference $|x| - p(x)$ at 41 equally spaced points. Then do the same with the 11 nodes $\cos((2i + 1)\pi/22)$, $i \in [\![0, 10]\!]$. Compare.

9. Prove or disprove: If n is a divisor of m, then each zero of T_n is a zero of T_m.

Pledged

Honor code: I have neither given nor received help on this test.