

## Math 170: Refresher

1. What is  $\frac{d^{15} \sin(x)}{dx^{15}}$ ?
2. At which points in  $[0, \pi]$  does the  $\cos$  function has a horizontal tangent line? a vertical tangent line?
3. Differentiate and integrate the expression  $\frac{1}{4x^2 + 1}$ .
4. Use the definition of derivative to establish that  $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$ .
5. Find the partial fraction decomposition for  $\frac{1}{x^2 - 1}$ .
6. Determine the following integrals:
  - $\int \frac{\tan(\sqrt{x})}{\sqrt{x}} dx$
  - $\int \ln(u) du$
  - $\int \arctan(x) dx$
  - $\int x \exp(-x^2) dx$
  - $\int \frac{dx}{x^4 + 6x^2 + 5} dx$
  - $\int \frac{x^2 - x}{1 - x^3} dx$
  - $\int \sinh^5(x) \cosh(x) dx$
  - $\int_0^\pi \cos^2(x) dx$

7. The improper integral  $\int_1^\infty \frac{dx}{x^p}$  converges for  $p$  and diverges for  $p$ .

8. Give alternate expressions for the following quantities:

$$\begin{aligned} \cos(\theta + \pi/2) &= \dots & \sin(\theta + \pi/2) &= \dots \\ \sin(2\theta) &= \dots & \cos(2\theta) &= \begin{cases} \dots \\ \dots \\ \dots \end{cases} \end{aligned}$$

9. Find the limits, if they exist,

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{\ln(x)} \quad \text{and} \quad \lim_{x \rightarrow -\infty} \frac{\sqrt{x}}{\ln(x)}$$

10. Express the hyperbolic sine function only in terms of exponential functions:

$$\sinh(x) =$$

11. What is the 'hyperbolic analog' of the formula  $\cos^2(x) + \sin^2(x) = 1$ ?

12. Are the following functions periodic, even/odd, increasing/decreasing, concave up/down?

$$\cos, \quad \exp, \quad \ln, \quad \cot$$

13. What is the area enclosed by the curves of equations  $y = x^2$  and  $y = \sqrt{x}$ ?

What is the area 'under' the curve of equation  $y = \sqrt{1 - x^2}$ ?

14. Determine the following limits:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}, \quad \lim_{x \rightarrow \pi/2^-} \frac{x}{\cos x}, \quad \lim_{x \rightarrow 0} \frac{4x}{\tan(7x)}.$$

15. Differentiate the following expressions with respect to  $x$ :

- $\sin(\ln(x^2))$
- $\tanh^3(\exp(-x^2))$

16. Find a formula for the inverse function  $f^{-1}$  of the function  $f$  given by

$$f(x) = \frac{1 - \sqrt{x}}{1 + \sqrt{x}}.$$